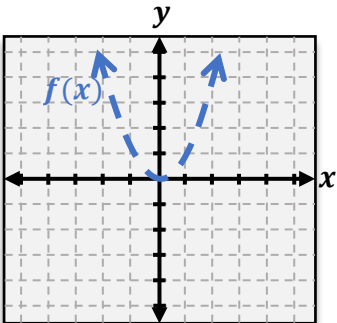
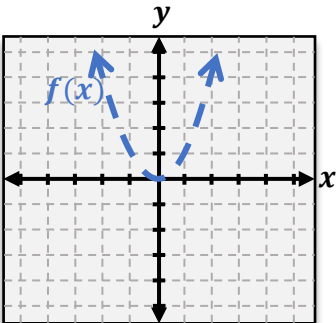
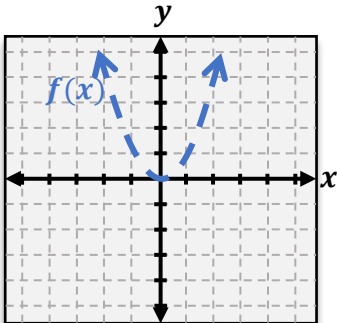


TOPIC: TRANSFORMATIONS

Intro to Transformations

- Transformations occur when a function is _____ and changes position and/or shape.

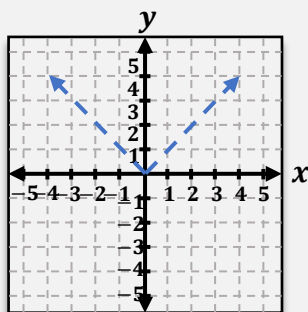
Transformations of Functions		
1) Reflection (FOLDING a function)	2) Shift (MOVING a function)	3) Stretch (SQUEEZING a function)
 $-f(x)$	 $f(x-h) + k$	 $c \cdot f(x)$

EXAMPLE: The graph of the $f(x) = |x|$ is shown below. Match the following functions $p(x)$, $q(x)$, & $r(x)$ to their corresponding graphs.

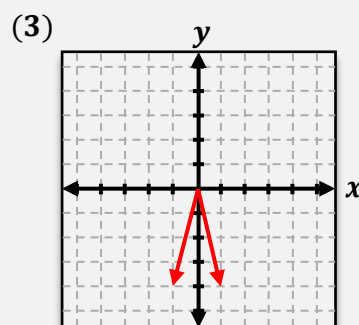
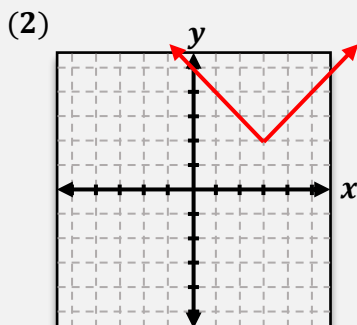
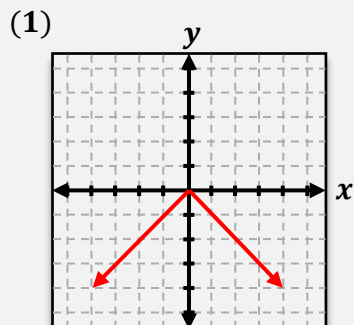
(A) $p(x) = |x - 3| + 2$

(B) $q(x) = -|x|$

(C) $r(x) = -4|x|$



Reflection	Shift	Stretch
$-f(x)$	$f(x-h) + k$	$c \cdot f(x)$



TOPIC: TRANSFORMATIONS

Reflections

Transformations of Functions

Reflection

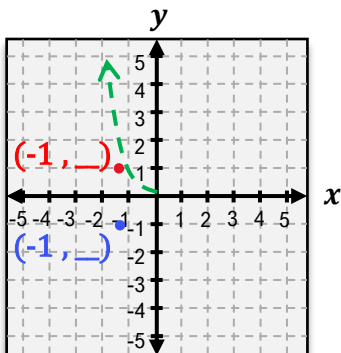
Shift

Stretch

- A reflection is a transformation where the function appears to be “_____” over the ____ -axis, or the ____ -axis.

REFLECTION OVER [X-AXIS | Y-AXIS]

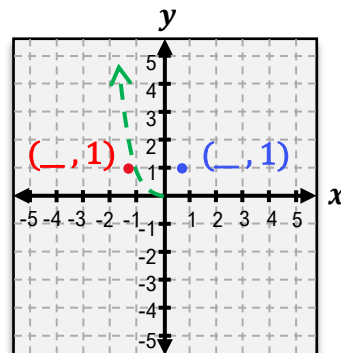
[x's | y's] change signs



$$f(x) \rightarrow -f(x)$$

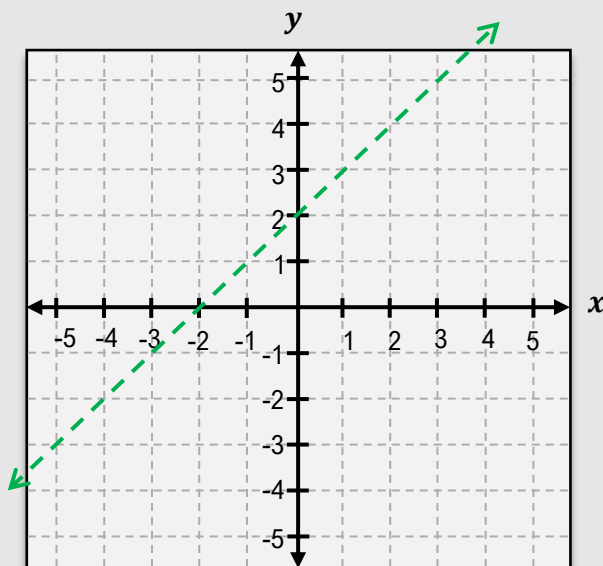
REFLECTION OVER [X-AXIS | Y-AXIS]

[x's | y's] change signs



$$f(x) \rightarrow f(-x)$$

EXAMPLE: Given $f(x) = x + 2$, let $h(x)$ be the function $f(x)$ with a **reflection** over the **x-axis**. Sketch the graph of $h(x)$, and determine which of the functions below matches the new function $h(x)$.



(A)

$$h(x) = \sqrt{x}$$

(B)

$$h(x) = \sqrt{-x}$$

(C)

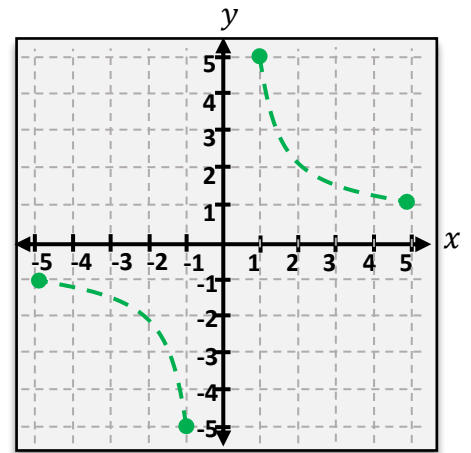
$$h(x) = -\sqrt{x}$$

(D)

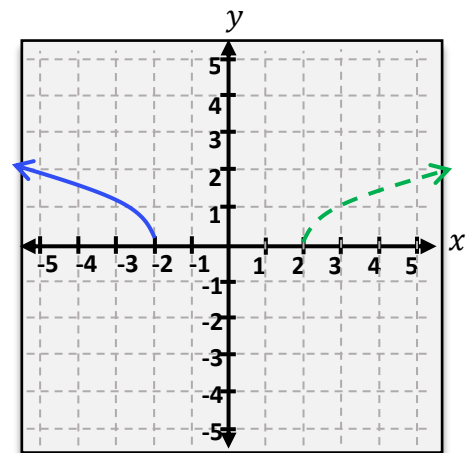
$$h(x) = -\sqrt{-x}$$

TOPIC: TRANSFORMATIONS

PROBLEM: Written below is a graph of the function $f(x)$. Sketch a graph of the function $g(x)$, where $g(x)$ is the function $f(x)$ after it has been reflected over the x -axis.



PRACTICE: Written below is a graph of the function $f(x) = \sqrt{x-2}$. If $g(x)$ is a reflection of $f(x)$ about the y -axis what is the equation for $g(x)$?



TOPIC: TRANSFORMATIONS

Shifting A Function

Transformations of Functions

Reflection

Shift

Stretch

- A shift occurs when a function is moved _____ and/or _____ from its original position.

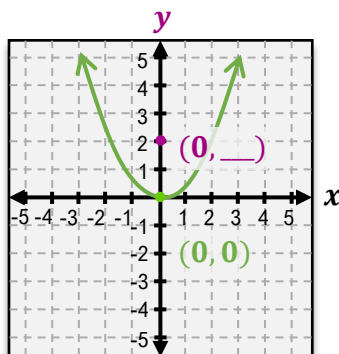
SHIFTS

General Form: $f(x) \rightarrow f(x - h) + k$

VERTICAL SHIFT ($h = 0$)

Vertical shifts change the
[x's | y's]

$$f(x) \rightarrow f(x - h) + k$$



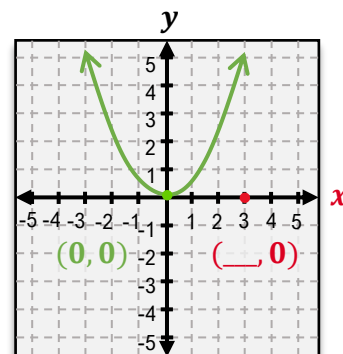
For $f(x) + k$, graph shifts [UP | DOWN]

For $f(x) - k$, graph shifts [UP | DOWN]

HORIZONTAL SHIFT ($k = 0$)

Horizontal shifts change the
[x's | y's]

$$f(x) \rightarrow f(x - h) + k$$



For $f(x + h)$, graph shifts [RIGHT | LEFT]

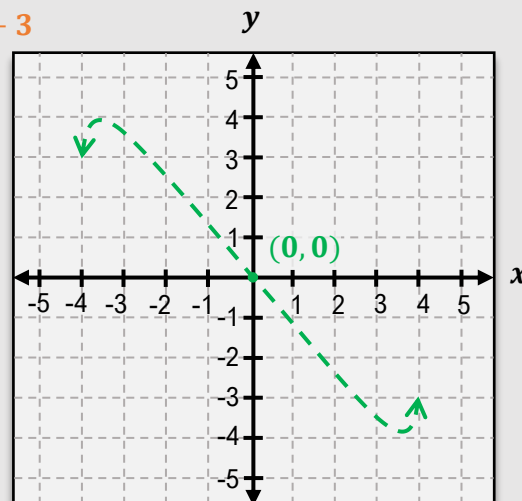
For $f(x - h)$, graph shifts [RIGHT | LEFT]

EXAMPLE: Given the function $f(x)$, sketch the transformation: $f(x - 2) + 3$

$$f(x) \rightarrow f(x - h) + k$$

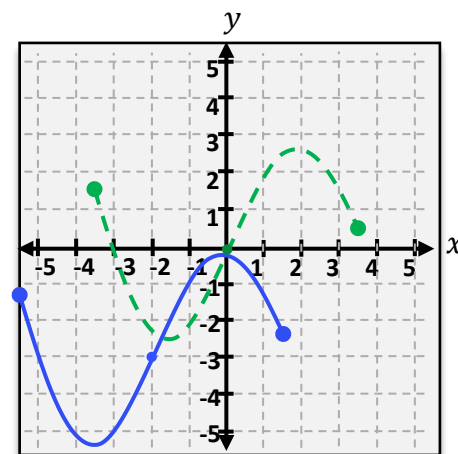
$h =$ _____

$k =$ _____



TOPIC: TRANSFORMATIONS

PRACTICE: The **green dotted line** in the graph below represents the function $f(x)$. The **blue solid line** represents the function $g(x)$, which is the function $f(x)$ after it has gone through a shift transformation. Find the equation for $g(x)$.



TOPIC: TRANSFORMATIONS

Graphs Of Shifted & Reflected Functions

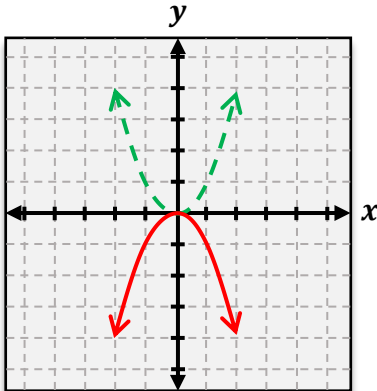
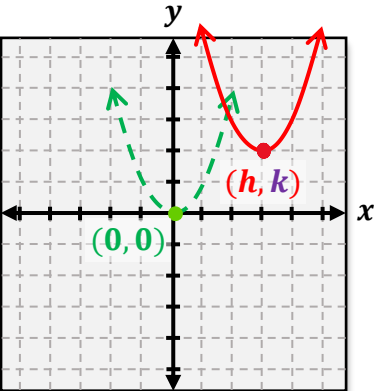
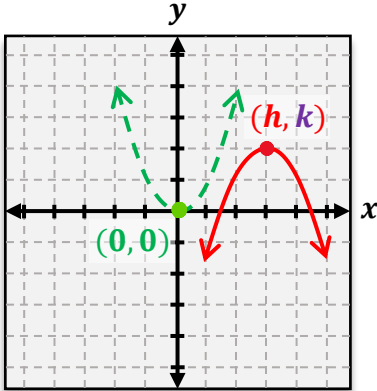
Transformations of Functions

Reflection

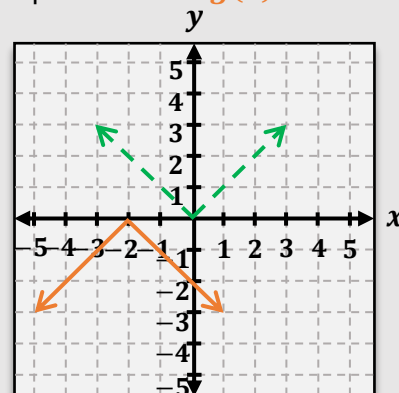
Shift

Stretch

- It is possible to see a _____ of transformations to a single function.

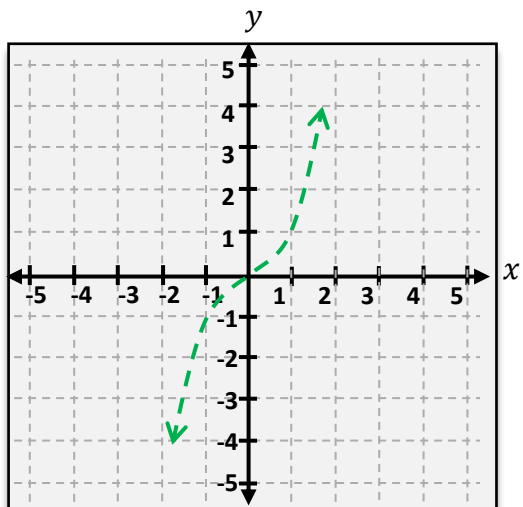
REFLECTION	SHIFTS	REFLECTIONS & SHIFTS
 $f(x) \rightarrow -f(x)$	 $f(x) \rightarrow f(x - h) + k$	 $f(x) \rightarrow -f(x - h) + k$

EXAMPLE: If $g(x)$ is a transformation of the function $f(x) = |x|$, write the equation for the $g(x)$.



TOPIC: TRANSFORMATIONS

PROBLEM: If $h(x)$ is a transformation of $f(x) = x^3$ where $f(x)$ is reflected over the x -axis, then shifted down 2 units. (A) write the equation for $h(x)$. (B) Sketch a graph of $h(x)$.



TOPIC: TRANSFORMATIONS

Stretches And Shrinks

Transformations of Functions

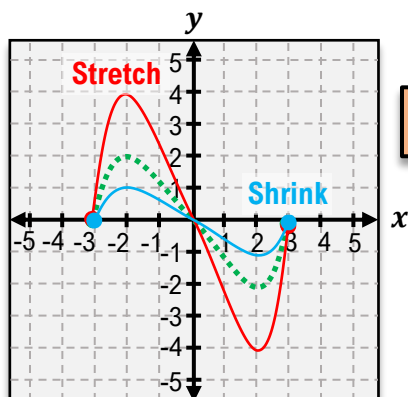
Reflection

Shift

Stretch

- Stretches/Shrinks occur when a _____ (c) is multiplied inside or outside the function.

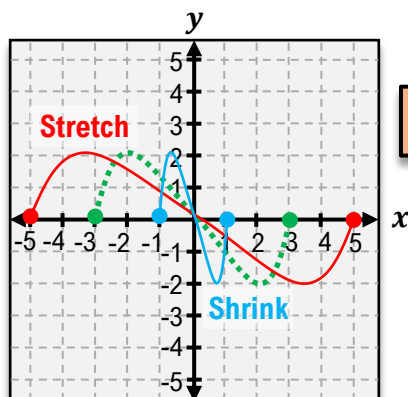
Vertical Stretch/Compress



$$f(x) \rightarrow c \cdot f(x)$$

- When $0 < c < 1$ the graph vertically [**STRETCHES** | **SHRINKS**]
- When $c > 1$ the graph vertically [**STRETCHES** | **SHRINKS**]

Horizontal Stretch/Compress



$$f(x) \rightarrow f(c \cdot x)$$

- When $0 < c < 1$ the graph horizontally [**STRETCHES** | **SHRINKS**]
- When $c > 1$ the graph horizontally [**STRETCHES** | **SHRINKS**]

EXAMPLE: Graphed below is the function $f(x)$. Sketch the graphs of the following functions:

(A)

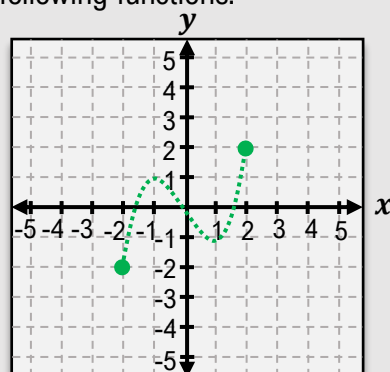
$$2 \cdot f(x)$$

(B)

$$\frac{1}{2} \cdot f(x)$$

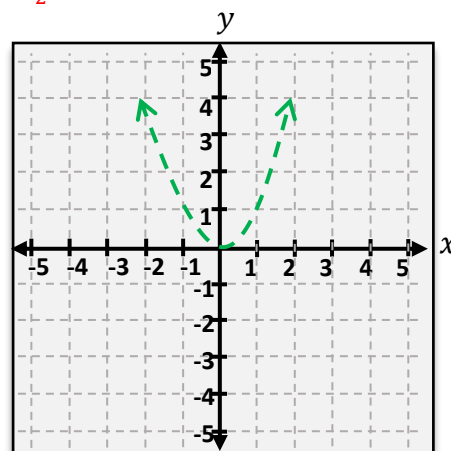
(C)

$$f\left(\frac{1}{2} \cdot x\right)$$



TOPIC: TRANSFORMATIONS

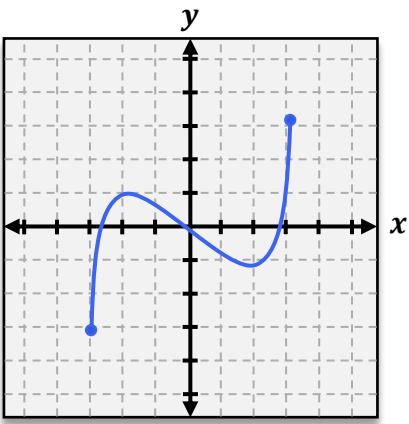
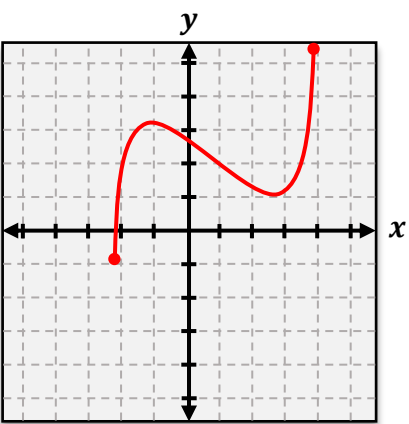
PROBLEM: Given that $f(x) = (c \cdot x)^2$, graph $f(x)$ when (A) $c = 2$, (B) $c = \frac{1}{2}$



TOPIC: TRANSFORMATIONS

Domain & Range of Transformed Functions

- A transformation can _____ the domain and range of a function.

Function: $f(x)$	Function: $f(x - 1) + 2$
	
Domain $\rightarrow D: [-3, 3]$ Range $\rightarrow R: [\text{____}, \text{____}]$	Domain $\rightarrow D: [-2, 4]$ Range $\rightarrow R: [\text{____}, \text{____}]$

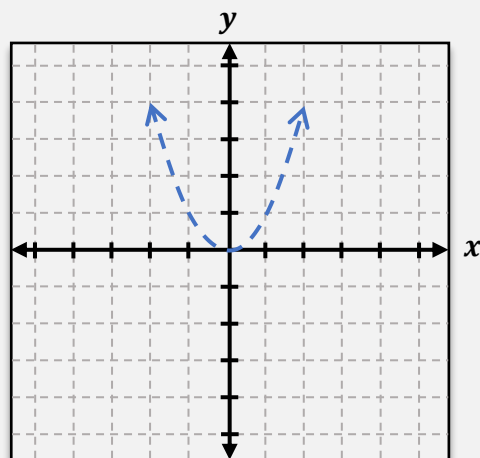
EXAMPLE: Given the function $f(x) = x^2$, sketch a graph of the function $g(x) = (x - 3)^2 + 2$ and determine its domain and range.

Domain Of $f(x)$: $(-\infty, \infty)$

Range Of $f(x)$: $[0, \infty)$

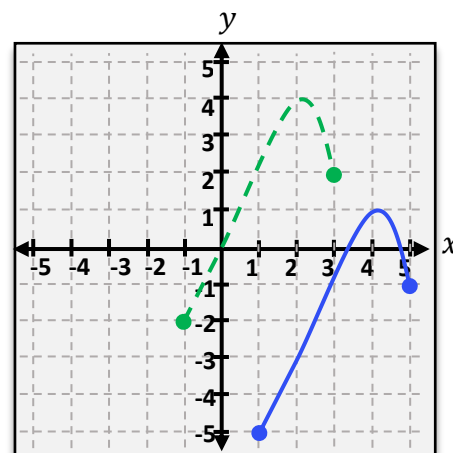
Domain Of $g(x)$:

Range Of $g(x)$:



TOPIC: TRANSFORMATIONS

PRACTICE: The green dotted curve below is a graph of the function $f(x)$. Find the domain and range of $g(x)$ (the blue solid curve), which is a transformation of $f(x)$.



Domain of $f(x) = [-1, 3]$

Range of $f(x) = [-2, 4]$