

## TOPIC: HYPOTHESIS TESTING USING CRITICAL VALUES

### Hypothesis Test: Critical Values & the Rejection Region

◆ A **critical value** (found from  $\alpha$ ) is the \_\_\_\_\_ between an "expected" test statistic & an "unusual" one.

#### P-value

$z$  or  $t \rightarrow$  find  $P$ -val  $\rightarrow$  compare to  $\alpha$

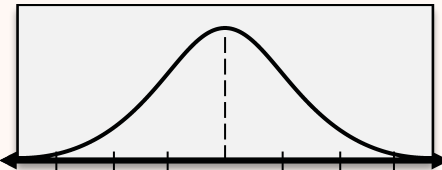
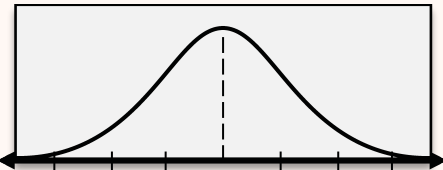
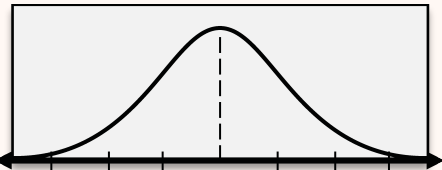
#### Critical Value

$\alpha \rightarrow$  find Critical Value  $\rightarrow$  compare to  $z$  or  $t$

► If the test statistic lies in the **rejection region** (area \_\_\_\_\_ critical value), then we *reject*  $H_0$ .

### EXAMPLE

You're a researcher looking at how the current mean age of students in your university has changed from last year, when the mean age was 23. So, you get a sample of students & calculate a test statistic. In each of the following situations, use  $\alpha = 0.05$  to find the critical value & determine if the given test stat ( $z$ ) is in the rejection region.

New <span style="float: right;">Hypothesis Test</span>		
Left Tail	Two Tail	Right Tail
<p>"Students now are <b>younger</b>..."</p> <p><math>H_0: \mu = 23; H_a: \mu &lt; 23</math></p>  <p>Critical Val(s): <math>z_\alpha =</math></p> <p>Test Stat: <math>z = -2.00</math></p> <p><math>z</math> [ IN   NOT IN ] Rejection Region [ REJECT   FTR ] <math>H_0</math></p>	<p>"Students now have a <b>different</b> mean age..."</p> <p><math>H_0: \mu = 23; H_a: \mu \neq 23</math></p>  <p>Critical Val(s): <math>-z_{\alpha/2} =</math>                      <math>z_{\alpha/2} =</math></p> <p>Test Stat: <math>z = -1.00</math></p> <p><math>z</math> [ IN   NOT IN ] Rejection Region [ REJECT   FTR ] <math>H_0</math></p>	<p>"Students now are <b>older</b>..."</p> <p><math>H_0: \mu = 23; H_a: \mu &gt; 23</math></p>  <p>Critical Val(s): <math>z_\alpha =</math></p> <p>Test Stat: <math>z = 0.8</math></p> <p><math>z</math> [ IN   NOT IN ] Rejection Region [ REJECT   FTR ] <math>H_0</math></p>

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### PRACTICE

Mark 'TRUE' or 'FALSE' for each of the following.

(A) The test statistic & the critical value are the same thing.

[ TRUE | FALSE ]

(B) The critical value is the boundary of the rejection region.

[ TRUE | FALSE ]

(C) You should always reject  $H_0$  if the test statistic is greater than the critical value.

[ TRUE | FALSE ]

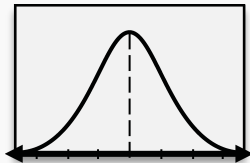
### PRACTICE

Use  $\alpha$  to find the critical value(s), then determine if the given test statistic is in the rejection region.

(A)

$$H_0: \mu = 7.5; \quad H_a: \mu > 7.5$$

$$\alpha = 0.01; \quad z = 2.17$$



Test is [ LEFT | TWO | RIGHT ] -tailed

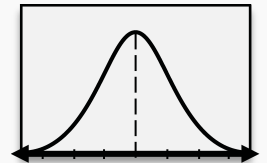
Critical Value(s):

Test stat [ IN | NOT IN ] rejection region.

(B)

$$H_0: p = 0.64; \quad H_a: p < 0.64$$

$$\alpha = 0.10; \quad z = -1.53$$



Test is [ LEFT | TWO | RIGHT ] -tailed

Critical Value(s):

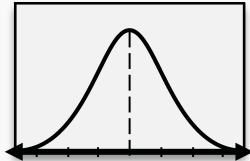
Test stat [ IN | NOT IN ] rejection region.

(C)

$$H_0: \mu = 98.6; \quad H_a: \mu \neq 98.6$$

$$\alpha = 0.05$$

$$t = 2.41; \quad df = 17$$



Test is [ LEFT | TWO | RIGHT ] -tailed

Critical Value(s):

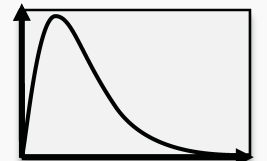
Test stat [ IN | NOT IN ] rejection region.

(D)

$$H_0: \sigma^2 = 0.3; \quad H_a: \sigma^2 > 0.3$$

$$\alpha = 0.05$$

$$\chi^2 = 61.7; \quad df = 51$$



Test is [ LEFT | TWO | RIGHT ] -tailed

Critical Value(s):

Test stat [ IN | NOT IN ] rejection region.

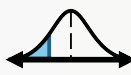
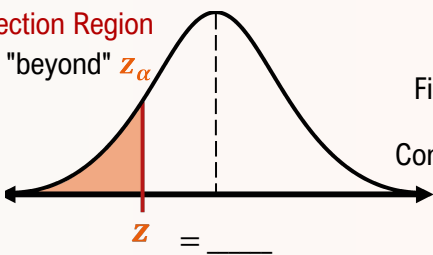
## TOPIC: HYPOTHESIS TESTING USING CRITICAL VALUES

### Hypothesis Test for Mean Using the Critical Value

◆ The **critical value method** for hyp. tests uses all the same steps as the *P*-value method, except step \_\_\_\_\_.

#### EXAMPLE

A soda company claims its cans contain 355 mL on average, but a sample of 50 cans has a mean of 352 mL. Test whether the mean volume is less than claimed using  $\alpha = 0.01$  &  $\sigma = 5$  mL.

Recall	P-Value Method	New	Critical Value Method
1) Hyp	$H_0: \mu = \underline{\hspace{2cm}}$ $H_a: \mu \underline{\hspace{2cm}}$		
2) Test Stat	$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ $\bar{x} = \underline{\hspace{2cm}}$ $\sigma = \underline{\hspace{2cm}}$ $n = \underline{\hspace{2cm}}$ $z = \underline{\hspace{2cm}}$		
3) P-Value	<div style="display: flex; justify-content: space-between;"> <div style="width: 30%;"> <p>Test Stat ↓ Find P-value from test stat ↓ Compare P-val to <math>\alpha</math></p>  <p>Area "beyond" <math>z</math>    <math>P\text{-value} = \underline{\hspace{2cm}}</math></p> </div> <div style="width: 35%; text-align: center;"> <p>3) Critical Value</p>  </div> <div style="width: 30%;"> <p>Test Stat ↓ Find Critical Value from <math>\alpha</math> ↓ Compare Test stat to Crit Val</p> </div> </div>		
4) Conclusion	<p>Because <math>P\text{-value}</math> [ &lt;   &gt; ] <math>\alpha</math>,</p> <p>We [ <b>REJECT</b>   <b>FAIL TO REJECT</b> ] <math>H_0</math>. There is [ <b>ENOUGH</b>   <b>NOT ENOUGH</b> ] evidence that <math>\mu &lt; 355</math>.</p>		
Criteria	$X$ is Normally Distributed? <input type="checkbox"/> <b>OR</b> $n > 30$ ? <input type="checkbox"/>		

## TOPIC: HYPOTHESIS TESTING USING CRITICAL VALUES

### PRACTICE

A soup company claims that the average sodium content of their most popular soup is 500 mg per can. A nutritionist collects a sample of 36 cans with mean sodium content 507 mg. Assume a known pop. standard deviation of 15 mg & test the nutritionist's suspicion that the mean sodium content is more than 500 mg using the critical value method with  $\alpha = 0.01$ .

$$H_0: \mu = \underline{\hspace{1cm}} \quad H_a: \mu = \underline{\hspace{1cm}}$$

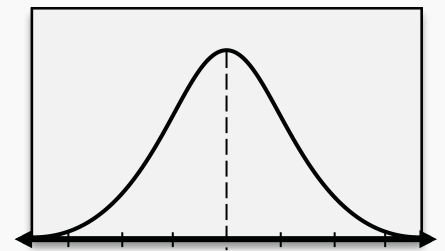
$$\bar{x} = \underline{\hspace{1cm}} \quad \sigma = \underline{\hspace{1cm}} \quad n = \underline{\hspace{1cm}}$$

$$z = \underline{\hspace{1cm}}$$

Critical Value(s):

Recall

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$



Because test stat. ( $z$ ) is [ **INSIDE** | **OUTSIDE** ] rejection region, we [ **REJECT** | **FAIL TO REJECT** ]  $H_0$ .

There is [ **ENOUGH** | **NOT ENOUGH** ] evidence to conclude that...

### EXAMPLE

A city claims that 40% of households have backyard gardens. A researcher surveys 150 households and finds 72 with gardens. Use  $\alpha = 0.10$  & the critical value method test if the true proportion is different from 40%.

$$H_0: p = \underline{\hspace{1cm}} \quad H_a: p = \underline{\hspace{1cm}}$$

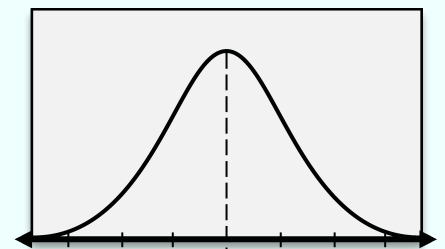
$$x = \underline{\hspace{1cm}} \quad n = \underline{\hspace{1cm}} \quad \hat{p} = \underline{\hspace{1cm}}$$

$$z = \underline{\hspace{1cm}}$$

Critical Value(s):

Recall

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$



Because test stat. ( $z$ ) is [ **INSIDE** | **OUTSIDE** ] rejection region, we [ **REJECT** | **FAIL TO REJECT** ]  $H_0$ .

There is [ **ENOUGH** | **NOT ENOUGH** ] evidence to conclude that...