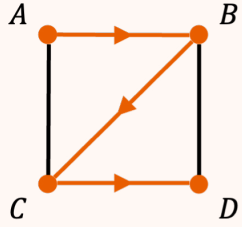
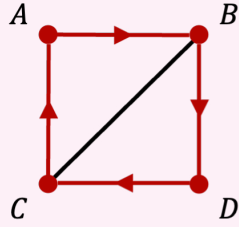


TOPIC: HAMILTON PATHS AND HAMILTON CIRCUITS

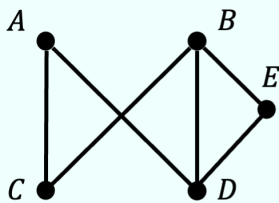
Intro to Hamilton Paths and Hamilton Circuits

- ◆ Recall: An **Euler path** crosses each *edge* exactly once. A **Hamilton Path** visits each _____ exactly once.
- ▶ A **Hamilton Circuit** starts & ends at the *same* vertex and visits every *other* vertex in the graph exactly once.

Hamilton Path	Hamilton Circuit
	

EXAMPLE

Use trial & error to find a Hamilton circuit in the graph below.

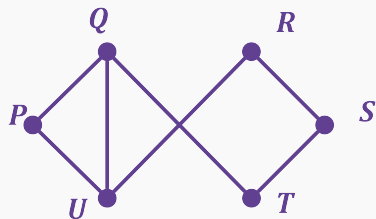


TOPIC: HAMILTON PATHS AND HAMILTON CIRCUITS

PRACTICE

Using the graph below, determine if the sequence of vertices describes a Hamilton path (H.P), a Hamilton circuit (H.C.), or neither.

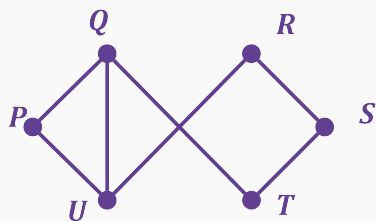
(A)



Q, U, P, Q, T, S, R

[H.P. | H.C. | NEITHER]

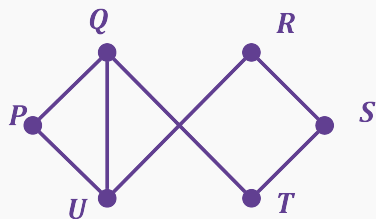
(B)



P, Q, T, S, R, U, P

[H.P. | H.C. | NEITHER]

(C)



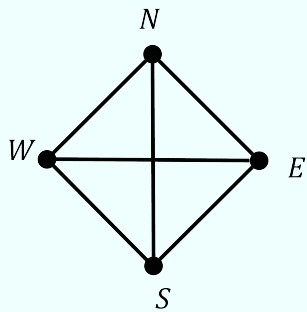
Q, P, U, R, S, T

[H.P. | H.C. | NEITHER]

TOPIC: HAMILTON PATHS AND HAMILTON CIRCUITS

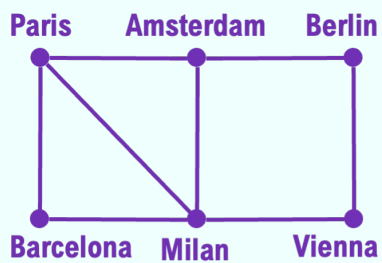
EXAMPLE

Use trial & error to find (**A**) a Hamilton path from $N \rightarrow W$ & (**B**) a Hamilton circuit starting at S .



EXAMPLE

The graph below represents train routes between major European cities. Use trial & error to find a route starting & ending in Milan, visiting each city exactly once.



TOPIC: HAMILTON PATHS AND HAMILTON CIRCUITS

Number of Hamilton Circuits in a Complete Graph

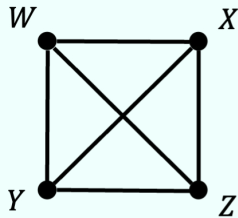
◆ A graph is **complete** if every _____ of vertices is connected by an edge.



► If a graph is complete, we can find how many Hamilton circuits it has using the **number of vertices, n** .

EXAMPLE

Determine the number of Hamilton Circuits in the graph below.



Complete? ☐

of vertices: $n = \underline{\hspace{2cm}}$

of Hamilton circuits: $\underline{\hspace{2cm}}$

New
$\frac{(\quad)!}{2}$

Recall

$$x! = x(x-1)(x-2) \cdots (2)(1)$$

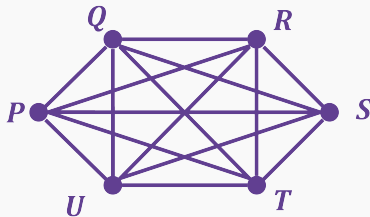
(Factorial)

◆ Note: Some textbooks consider going in the opposite direction to be a new circuit & use the alt formula $(n-1)!$

PRACTICE

Determine if the graph must have Hamilton circuits. If so, how many?

Note: We will be using the formula below.



Complete? ☐

of vertices: $n = \underline{\hspace{2cm}}$

of Hamilton circuits: $\underline{\hspace{2cm}}$

Recall
$\frac{(n-1)!}{2}$

TOPIC: HAMILTON PATHS AND HAMILTON CIRCUITS

PRACTICE

How many Hamilton circuits exist in a complete graph with the given number of vertices, n ?

Note: We will be using the formula below.

(A) $n = 3$

(B) $n = 12$

Recall

$$\frac{(n-1)!}{2}$$

2

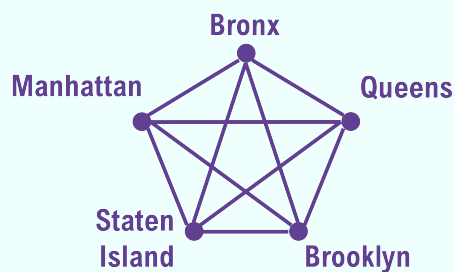
EXAMPLE

The graph below represents train routes between major European cities. Use trial & error to find a route starting & ending in Milan, visiting each city exactly once.

Recall

$$\frac{(n-1)!}{2}$$

2



TOPIC: HAMILTON PATHS AND HAMILTON CIRCUITS

Weighted Graphs

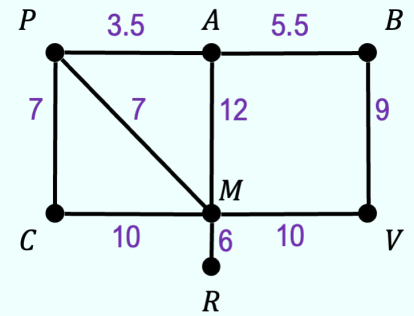
◆ In a **weighted graph**, edges have _____ (weights), which often represent costs, distances, or times.

EXAMPLE

The graph below represents train routes between major European cities. The weights represent the time of each train ride in hours. Find the total time (weight) of each path or circuit.

(A) Amsterdam to Milan
 $A - M$

(B) Amsterdam to Milan
 $A - P - C - M$

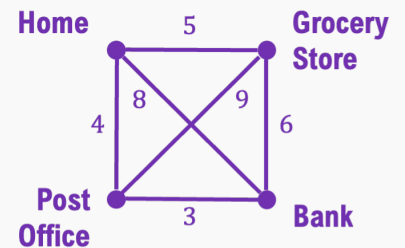


(C) Circuit starting & ending in Barcelona (C)
 $C - M - A - P - C$

PRACTICE

The weights on the graph below represent distances (in miles). Find the total distance (weight) of each path or circuit.

(A) Home, Grocery Store, Post Office, Bank



(B) Home, Grocery Store, Post Office, Bank, Home

TOPIC: HAMILTON PATHS AND HAMILTON CIRCUITS

EXAMPLE

The table shows flight costs between London, New York City, and Tokyo. Draw a weighted graph to represent these flights. Then find the total cost for a round trip, starting & ending in Tokyo.

Flight	Cost (\$)
LOL – NYC	550
LON – TOK	1000
NYC – TOK	1500

EXAMPLE

Each of the 5 boroughs of New York City are connected through public transit. The average time to travel between boroughs using public transit is given in the table.

(A) Draw a weighted graph to represent the 5 boroughs.

Boroughs	Time (min)
BX – QNS	75
BX – MAN	45
BX – BKN	70
BX – SI	100
QNS – MAN	45
QNS – BKN	60
QNS – SI	100
MAN – BKN	45
MAN – SI	70
BKN – SI	75

(B) Find a route visiting all 5 boroughs (Hamilton circuit).

(C) How long (in minutes) would your route take?

TOPIC: HAMILTON PATHS AND HAMILTON CIRCUITS

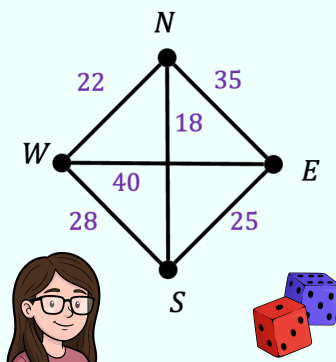
The Brute Force Method

◆ **Traveling Salesperson Problem:** A salesperson must travel to *each* city in a network *once* & *return* to the start.

► The **optimal solution** to the TSP is the Hamilton Circuit with the _____ total weight (cost, distance, time).

EXAMPLE

Colleen sells RPG dice sets at 4 markets across town every Saturday. Travel times are listed in minutes. Find a round trip starting at vertex *N*.



Circuit	Weight

HOW TO: Solve a TSP using Brute Force

- 1) Draw a graph to represent the problem
- 2) List _____ possible Hamilton Circuits
- 3) Find the total _____ of each HC
- 4) Pick the HC with the **lowest** total weight

EXAMPLE



Nick coaches wrestling at Maple Grove High School and is organizing a 3-day meet at three other schools. The team will start & end at MGHS to visit each school exactly once: Riverside, Summit, & Northgate. Driving times in hours between schools are listed in the table.



Note: We will be using the formula below.

(A) Explain why this is a TSP.

Recall

$$\frac{(n-1)!}{2}$$

Schools	Time
MG – R	2.5
MG – S	3
MG – N	4.5
R – S	3
R – N	2.5
S – N	4

(B) Find the route with the minimum total travel time.

TOPIC: HAMILTON PATHS AND HAMILTON CIRCUITS

EXAMPLE

Jenn runs a tutoring business in Dallas. She will fly to Boston University, UCLA, Chicago, and Georgia Tech to promote her services, then return to Dallas, visiting each school exactly once. The table gives one-way airfare in dollars between cities. Find a route that minimizes total airfare. *Note: We will be using the formula below.*

Cities	Airfare	Cities	Airfare
Dallas – Boston	200	Boston – LA	250
Dallas – LA	125	Boston – Chicago	115
Dallas – Chicago	140	Boston – Atlanta	195
Dallas – Atlanta	130	LA – Chicago	175
Chicago – Atlanta	150	LA – Atlanta	195

Recall

$$\frac{(n - 1)!}{2}$$

