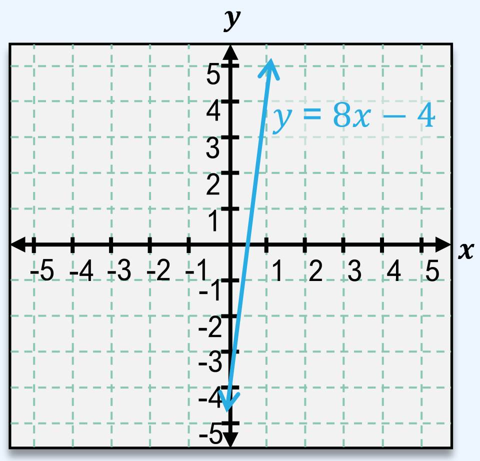
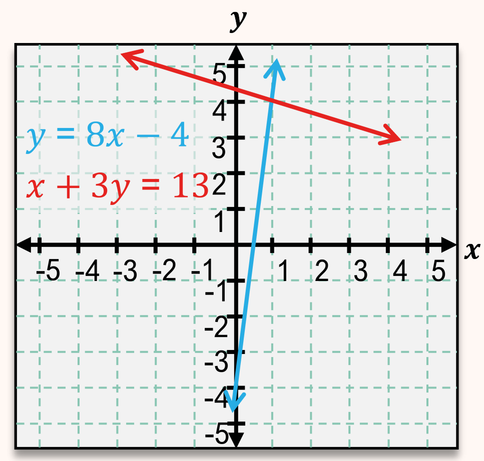
TOPIC: SYSTEMS OF LINEAR EQUATIONS

Introduction to Systems of Linear Equations

◆ Previously, we solved & plotted *single* equations on graphs. Now we'll look at **Systems of (____________) Equations**.

► To solve, find (x, y) which satisfy ______ equations. Graphically, this is where the lines ________________.

Recall Single Equation	**New** Multiple Equations a.k.a "System"
	

EXAMPLE

Determine if each point is a solution to the equation.

$(\boldsymbol{A})$ $(-2,0)$ $(\boldsymbol{B})$ $(0,-4)$ $(\boldsymbol{C})$ $(1,4)$

Solution: **[1 | MANY]** point(s)
satisfying **[1 | ALL]** line(s)

Determine if each point is a solution to the system of equations.

$(\boldsymbol{A})$ $(0,-4)$ $(\boldsymbol{B})$ $(-2,5)$ $(\boldsymbol{C})$ $(1,4)$

Solution: **[1* | MANY]** point(s)
satisfying **[1 | ALL]** line(s)

* True for ***most*** problems, but there are other types of solutions

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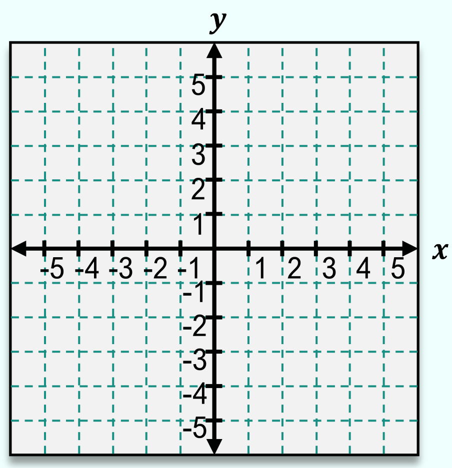
Solve a System of Linear Equations by Graphing

- To solve a system of equations graphically, plot the 2 lines ($y = mx + b$ form helps!) & find their ______________.
  - *Always* check the solution by plugging (x, y) of intersection into BOTH equations.

EXAMPLE Solve the system of equations by graphing.

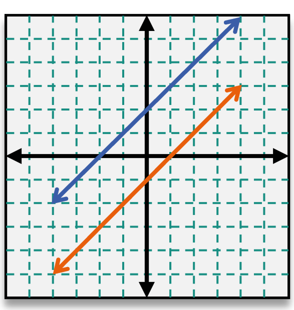
$$\begin{cases} y = x - 3 \\ 3x + y = 5 \end{cases}$$

Intersection Point: (__ , __)



- Most systems have 1 solution, but you might see…

Parallel Lines

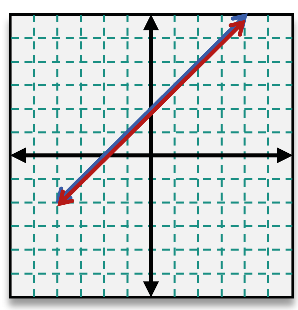


$y = x + 2$

$y = x - 1$

[0 | MANY] Solutions

Same Lines



$y = x + 2$

$2y = 2x + 4$

simplifies to

$y = x + 2$

[0 | MANY] Solutions

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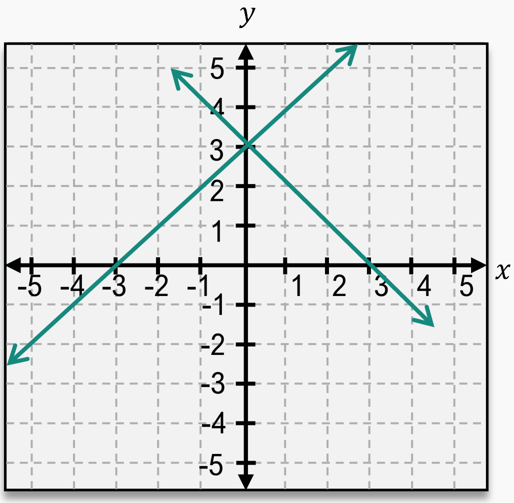
PRACTICE

Use the graph to identify the solution to the system of equations or identify that the system has 0 or infinitely many solutions. If there is a solution, check that the point satisfies both equations.

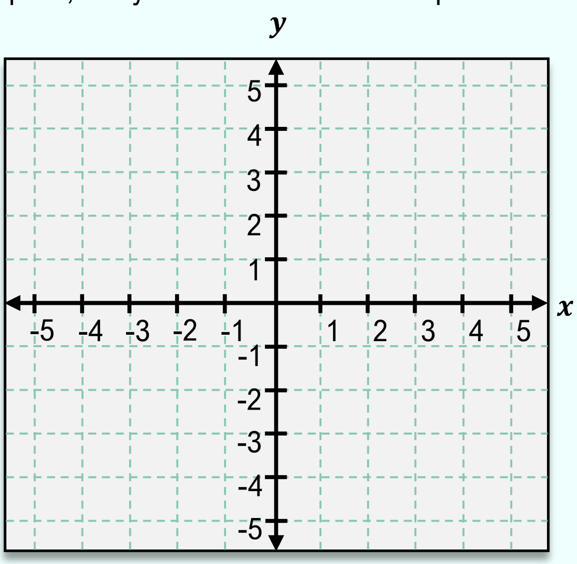


EXAMPLE

Graph the system of equations. Identify the intersection point, verify it is a solution to both equations.

$y = 2x + 3$

$y = x + 4$



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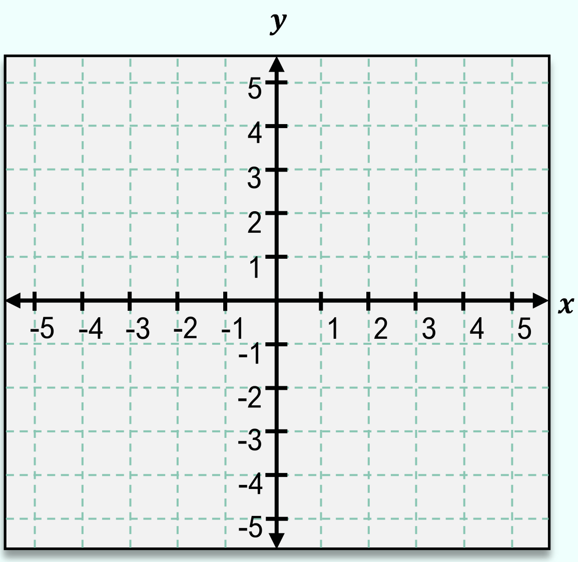
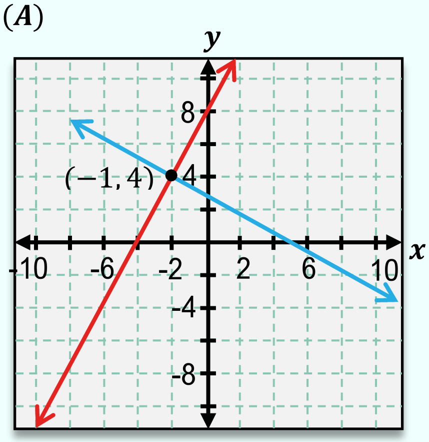
EXAMPLE Solve the system of equations by graphing.

$$\begin{cases} 6y + 2x = -6 \\ 3y + x = -3 \end{cases}$$

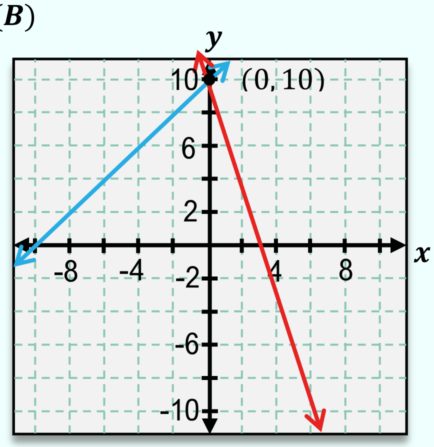


EXAMPLE Match each system of equations to its graph and solution.

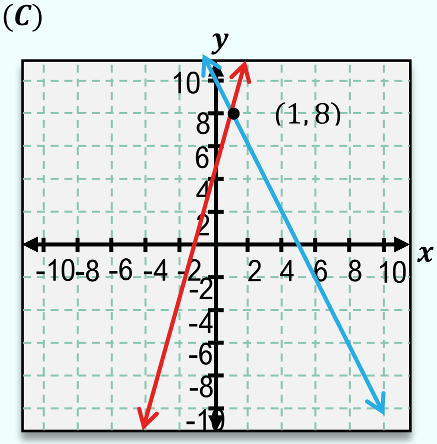
(*A*)



(*B*)



(*C*)



(1)

$$y = 3x + 5$$
$$y = -2x + 10$$

(2)

$$y = 4x + 8$$
$$2x + 3y = 10$$

(3)

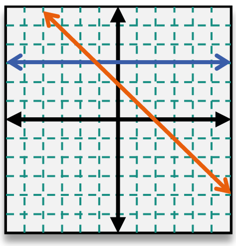
$$3x + y = 10$$
$$-x + y = 10$$

<u>TOPIC: SYSTEMS OF LINEAR EQUATIONS</u>

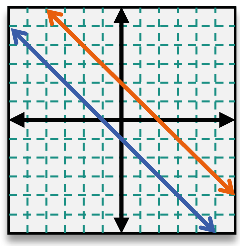
<u>Determining Number of Solutions in a System</u>

◆ Recall: There are **3 types** of system of equations based on the number of solutions they have.

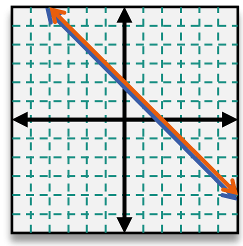
1 Solution



0 Solutions



Infinitely Many Solutions



◆ To find the # of solutions in a system *without graphing*, write equations as $y = mx + b$, ___________ m's & b's.
(Slope-Intercept Form)

EXAMPLE Determine how many solutions each system has.

$(\boldsymbol{A})$

$y = 3 \qquad x + y = 2$

$m = \qquad m =$

$b = \qquad b =$

Slopes are **[SAME | DIFF]**

[1 | 0 | ∞] solutions

"Consistent" & "*Independent*"

$(\boldsymbol{B})$

$y = -x - 1 \qquad x + y = 2$

$m = \qquad m =$

$b = \qquad b =$

Slopes are **[SAME | DIFF]**

y-ints are **[SAME | DIFF]**

[1 | 0 | ∞] solutions

"Inconsistent"

$(\boldsymbol{C})$

$-x - y = -2 \qquad x + y = 2$

$m = \qquad m =$

$b = \qquad b =$

Slopes are **[SAME | DIFF]**

y-ints are **[SAME | DIFF]**

[1 | 0 | ∞] solutions

"Consistent" & "*Dependent*"

TOPIC: SYSTEMS OF LINEAR EQUATIONS

PRACTICE Determine the number of solutions the system of equations has *without* graphing.

$(\boldsymbol{A})$

$$\begin{cases} y + 2x = 8 \\ y - 5x = -6 \end{cases}$$

(A) 1

(B) Infinitely Many

(C) 0

$(\boldsymbol{B})$

$$\begin{cases} y = \frac{1}{4}x + 11 \\ y = \frac{1}{4}x - 6 \end{cases}$$

(A) 1

(B) Infinitely Many

(C) 0

PRACTICE Without using a graph, determine the number of solutions in the following systems of equations, then classify the system.

$$\begin{cases} y + 2x = 8 \\ y - 5x = -6 \end{cases}$$

(A) Independent and Consistent

(B) Dependent and Consistent

(C) Inconsistent

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Solving Systems of Linear Equations - Substitution

◆ One way to solve systems *without* graphing is by ________________ one equation into another.

EXAMPLE Solve the system of equations using substitution.

$$y = 7x - 14$$
$$2x - y = 4$$

HOW TO: Solve Systems of Equations - Substitution

1) Choose easiest equation to isolate $\boldsymbol{x}$ or $\boldsymbol{y}$ as **(A)**

2) Solve **(A)** for $\boldsymbol{x}$ OR $\boldsymbol{y}$

3) Substitute **(A)** into **(B)**, then solve **(B)**

4) Plug value from **3)** back into *either* eq'n, then solve

5) Check answer by plugging values into both eqn's

PRACTICE Use subsitution to solve the following system of linear equations.

$(\boldsymbol{A})$
$$4x + y = 1$$
$$x - y = 4$$

$(\boldsymbol{B})$
$$4x + 2y = 7$$
$$x + 5y = 4$$

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Solving Systems of Linear Equations - Elimination

◆ Another way to solve systems of equations is by ____________ the equations & _______________ a variable.

► If not asked to use a specific method, use this when equations are in standard form or have large coefficients. $(\boldsymbol{Ax} + \boldsymbol{By} = \boldsymbol{C})$

Recall: Substitution	New: Elimination
$y = 5x - 3$ $x = 2$	$x + y = 1$ $-x + y = 5$

EXAMPLE Solve the system of equations using elimination.

$$3x + 2y = 1$$
$$-x + y = 3$$

HOW TO: Solve Systems of Equations - Elimination

1) Write BOTH equations in the form $\boldsymbol{Ax} + \boldsymbol{By} = \boldsymbol{C}$, aligning coeff's vertically on top of each other

2) Multiply eq'n(s) by # (+ or –) so $\boldsymbol{x}$ or $\boldsymbol{y}$ coeff's are __________ with _______________ signs

3) ________ equations vertically to eliminate one variable, then solve for other

4) Plug value from **3)** back into *either* eq'n, then solve

5) Check answer by plugging values into both eqn's

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How to Multiply Equations in Elimination Method

◆ To determine what # to multiply by in Step **2)**, look at the coefficients of each equation.

Elimination Method - What to Mutiply Equation(s) by to Eliminate Variable				
If coefficients of x or y are…	**Equal with *OPPOSITE* sign**	**Equal with *SAME* sign**	**Factors of each other (Evenly divisible)**	**Anything Else**
Multiply…	Nothing! Just add	Either eq'n by – 1	Eq'n with smaller coeff's by quotient	Each eq'n by *other* coeff (+ or –)
EXAMPLE	$7x+13y=12$ $-7x+2y=18$	$5x+7y=17$ $6x+7y=12$	$12x-5y=24$ $3x-2y=6$	$6x+2y=-10$ $-4x-3y=15$

HOW TO: Solve Systems of Equations - Elimination
1) Write BOTH equations in the form $\boldsymbol{Ax+By=C}$, aligning coeff's vertically on top of each other
2) Multiply eq'n(s) by # (+ or –) so $\boldsymbol{x}$ or $\boldsymbol{y}$ coeff's are **EQUAL** with **OPPOSITE** signs
3) **ADD** equations vertically to eliminate one variable, then solve for other
4) Plug value from **3)** back into *either* eq'n, then solve

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EXAMPLE Without *fully* solving, multiple one or both equation(s) by an appropriate factor to cancel out a variable.

Elimination Method - What to Mutiply Equation(s) by to Eliminate Variable				
If coefficients of x or y are…	**Equal with *OPPOSITE* sign**	**Equal with *SAME* sign**	**Factors of each other (Evenly divisible)**	**Anything Else**
Multiply…	Nothing! Just add	Either eq'n by – 1	Eq'n with smaller coeff's by quotient	Each eq'n by *other* coeff (+ or –)

$(\boldsymbol{A})$

$$\begin{aligned} 2x + 3y &= 1 \\ x - y &= 3 \end{aligned}$$

$(\boldsymbol{B})$

$$\begin{aligned} 5x + 3y &= 10 \\ -7x + 5y &= 15 \end{aligned}$$

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PRACTICE Use the elimination method to solve the following system of linear equations.

HOW TO: Solve Systems of Equations - Elimination

1) Write BOTH equations in the form $\boldsymbol{Ax} + \boldsymbol{By} = \boldsymbol{C}$, aligning coeff's vertically on top of each other

2) Multiply eq'n(s) by # (+ or –) so $\boldsymbol{x}$ or $\boldsymbol{y}$ coeff's are **EQUAL** with **OPPOSITE** signs

3) **ADD** equations vertically to eliminate one variable, then solve for other

4) Plug value from **3)** back into *either* eq'n, then solve

5) Check answer by plugging values into both eqn's

$(\boldsymbol{A})$

$$2x + y = 1$$
$$3x - y = 4$$

$(\boldsymbol{B})$

$$10x - 4y = 5$$
$$5x - 4y = 1$$