<u>**TOPIC: AREA UNDER A CURVE**</u>

Area Under Linear Functions

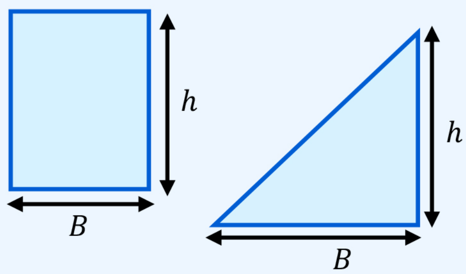
◆ You'll often need to find the "*area under the curve,*" or the area between the function's graph and the ____________.

- For linear functions, we can use the area of 1+ familiar shape(s), like *triangles* & *rectangles*.

EXAMPLE Given $f(x) = x$, find the area under the curve from $x = 0$ to $x = 5$.

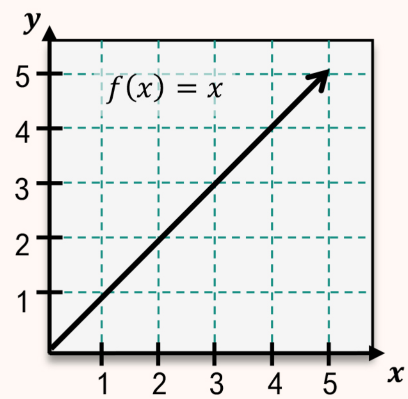
Recall — Area Of Basic Shapes

Area Of Rectangle$= B \cdot h$



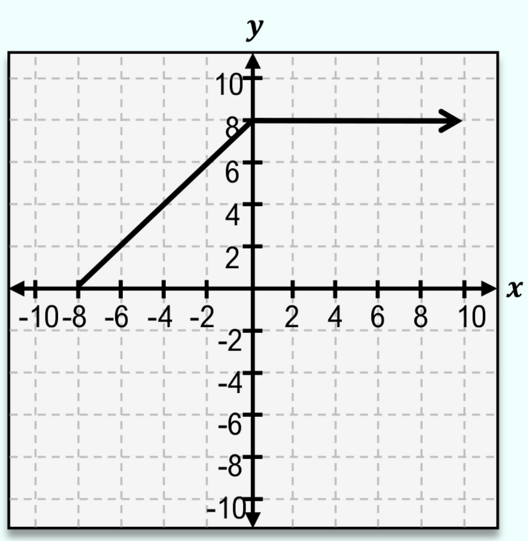
Area Of Triangle $= \frac{1}{2} B \cdot h$

New — Area Under A Curve

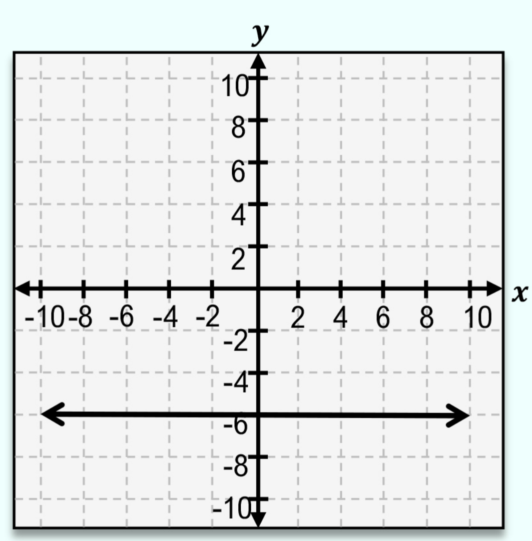


EXAMPLE Calculate the area under the curve for each interval in the functions below.

$(\boldsymbol{A})$ From $x = -8$ to $x = 4$



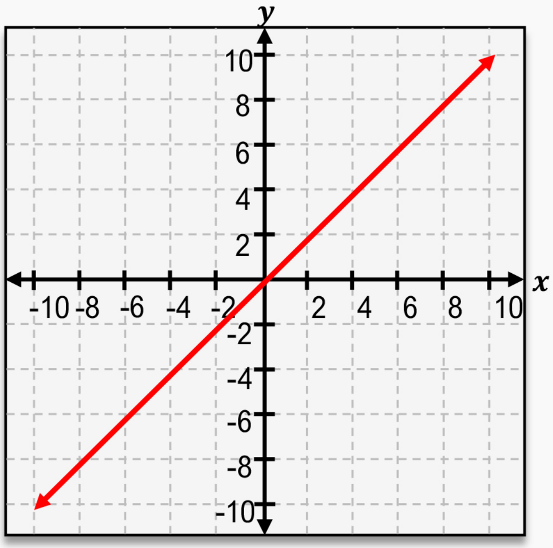
$(\boldsymbol{B})$ From $x = -6$ to $x = 8$



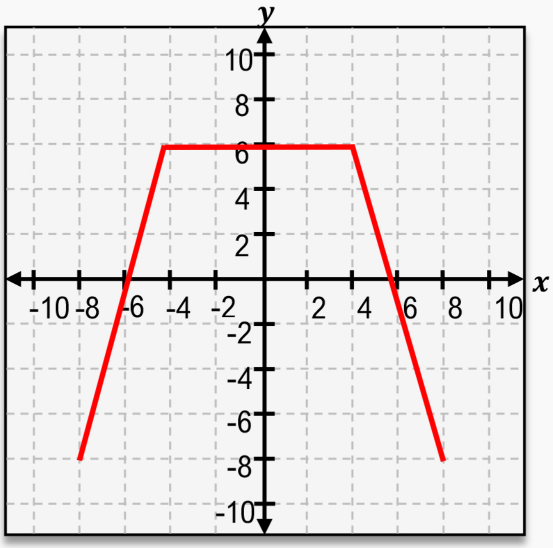
◆ "Under the curve" can be misleading. When fcns go *below* the x-axis, the area is "*above*" the curve & ____________.

TOPIC: AREA UNDER A CURVE

PRACTICE Find the area under the curve in the function below from $x = -4$ to $x = 8$.



PRACTICE Find the area under the curve in the function below from $x = -2$ to $x = 8$.



TOPIC: AREA UNDER A CURVE

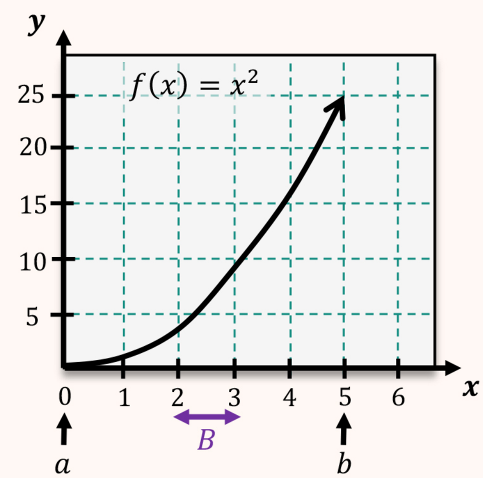
Approximating Area Under A Curve

- Recall: We used shapes to find areas under curves for *straight* lines, but that DOESN'T work for _________ graphs.
  - We can *approximate* this area by cutting it into many (n) rectangles w/ base B & height h, then _________.

EXAMPLE Use 5 rectangles to approximate the area under $f(x) = x^2$ from $x = 0$ to $x = 5$ with left and right endpoints.

Left Endpoint Approximation (L)	Right Endpoint Approximation (R)

Left Endpoint Approximation (L)



Recall

Area Of Rectangle = $B \cdot h$

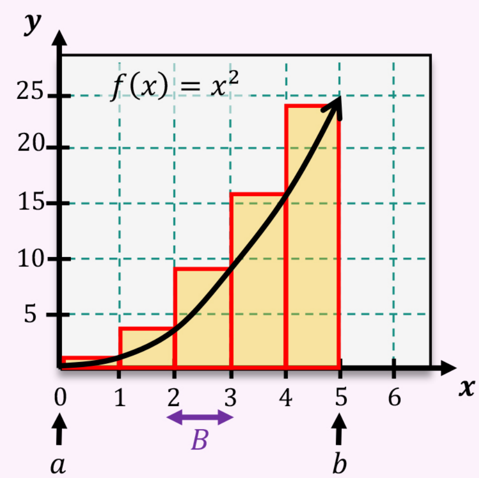
New

$A \approx$ ____________________,

where $B = \frac{b-a}{n}$ and $h_k = f(x_k)$

h = top-**[LEFT | RIGHT]** corner of each rectangle

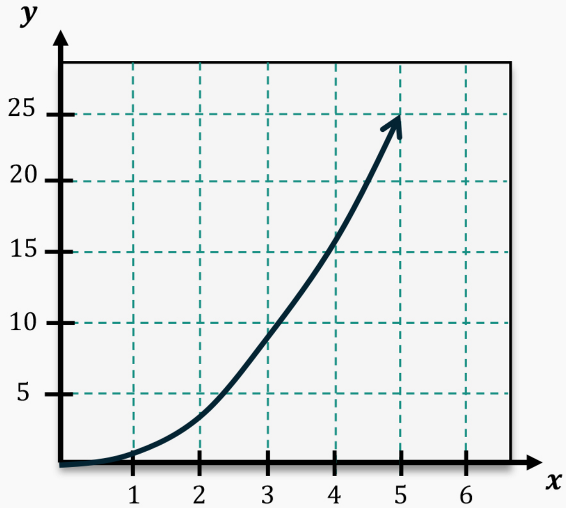
Right Endpoint Approximation (R)



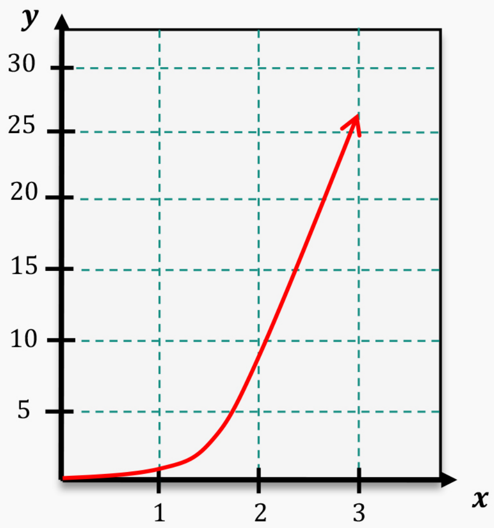
h = top-**[LEFT | RIGHT]** corner of each rectangle

TOPIC: AREA UNDER A CURVE

PRACTICE Use 3 rectangles to approximate the area under $f(x) = x^2$ from $x = 2$ to $x = 5$ with right endpoints.



PRACTICE Use 6 rectangles to approximate the area under $f(x) = x^3$ from $x = 0$ to $x = 3$ with left endpoints.



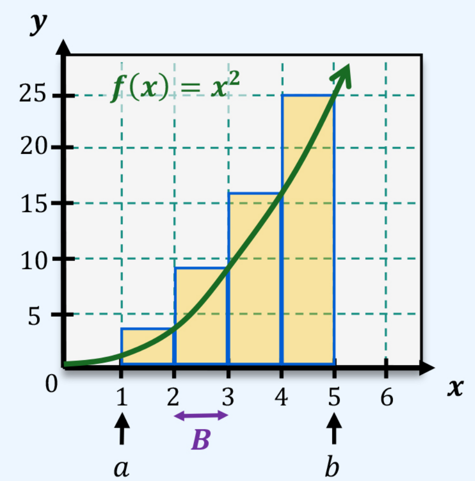
TOPIC: AREA UNDER A CURVE

Approximating Area Under A Curve Algebraically

◆ To approximate area without a graph, we'll still just add rectangles, but using a formula.

EXAMPLE Use 4 rectangles to approximate the area under $f(x) = x^2$ from $x = 1$ to $x = 5$ with right endpoints.

Recall | **Approx. Area Graphically**



$A \approx B \cdot h_1 + B \cdot h_2 \ldots + B \cdot h_n$

$= 1 \cdot 4 + 1 \cdot 9 + 1 \cdot 16 + 1 \cdot$

$= 55$

$h_1 = 4$

$h_2 = 9$

$h_3 = 16$

$h_4 = 25$

New | **Approx. Area Algebraically**

BASE HEIGHT

$$A \approx \sum_{k=1}^{n} \Delta x \cdot f(x_k)$$

$a =$ _____ (Starting x-value of interval)

$b =$ _____ (Ending x-value of interval)

$n =$ _____ (# of rectangles)

$\Delta x = \frac{b-a}{n} =$ _____ $=$ __ (rectangle base, constant)

$f(x_k)$ (rectangle height, for each k)

$k = 1, 2, \ldots, n$ (counter)

For left endpts: $x_1 = a +$ __ $\cdot \Delta x$, $x_2 = a + 1\Delta x$, $x_3 = a + 2\Delta x$ ...

For right endpts: $x_1 = a +$ __ $\cdot \Delta x$, $x_2 = a + 2\Delta x$, $x_3 = a + 3\Delta x$ ...

$f(x_1) = f($ __ $) =$ _____

$f(x_2) = f($ __ $) =$ _____

$f(x_3) = f($ __ $) =$ _____

$f(x_4) = f($ __ $) =$ _____

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EXAMPLE Use 4 rectangles to approximate the area under $f(x) = 4x^2$ from $x = 0$ to $x = 8$ with left endpoints.

Recall

$$A \approx \sum_{k=1}^{n} \Delta \boldsymbol{x} \cdot \boldsymbol{f}(\boldsymbol{x}_{\boldsymbol{k}})$$

$a =$ ______

$b =$ ______

$n =$ ______

$\Delta x = \frac{b-a}{n} =$ ______ $=$ _____

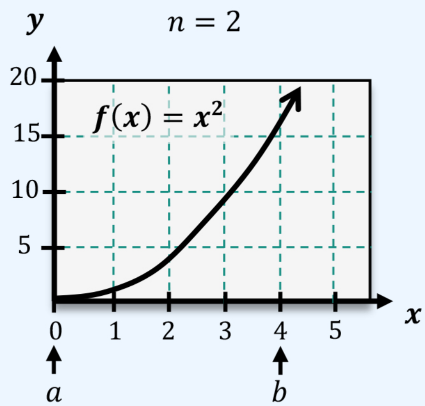
TOPIC: AREA UNDER A CURVE

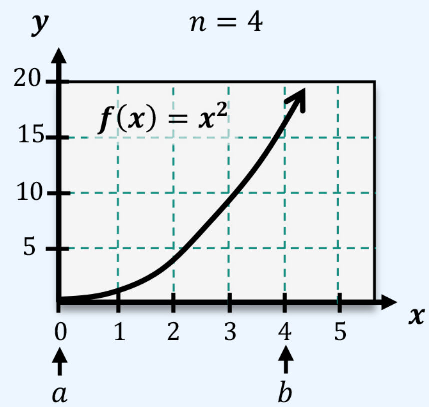
Using Limits to Find Exact Area

◆ Recall: We can *approximate* the area under the curve using rectangles.

- As we draw ***more*** rectangles (n), they get ***thinner*** (Δx), & the approximation gets more ____________!

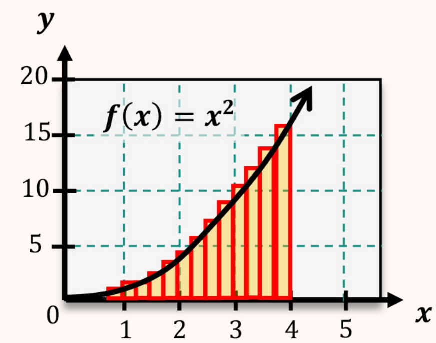
Recall	Approximate Area with Rectangles	New	Exact Area With Limits





$$A \approx \sum_{k=1}^{n} \Delta x \cdot f(x_k)$$

$$\Delta x = \frac{b-a}{n}$$



$$A \quad \lim_{n\to} \sum_{k=1}^{n} \Delta x \cdot f(x_k)$$

$$x_k = a + \_\_\_ \cdot \Delta x$$

EXAMPLE Find the exact area under $f(x) = \frac{1}{2}x$ from $x = 0$ to $x = 6$ using the limit definition.

$$A = \lim_{n\to\infty} \sum_{k=1}^{n}$$

$$A = \lim_{n\to\infty} \qquad \sum_{k=1}^{n}$$

Recall

$$\sum_{k=1}^{n} c = cn$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

TOPIC: AREA UNDER A CURVE

PRACTICE Find the area under the curve of the function $f(x) = 3x + 2$ from $x = 0$ to $x = 4$ using limits.

PRACTICE Find the area under the curve of the function $f(x) = x^2$ from $x = 0$ to $x = 8$ using limits.