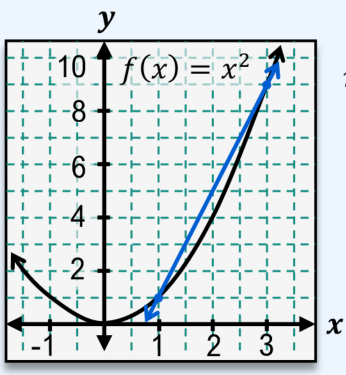
TOPIC: TANGENT LINES AND DERIVATIVES

Slopes of Tangent Lines

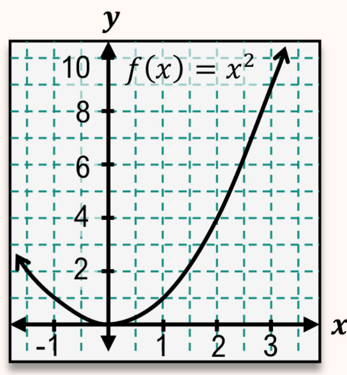
- Recall: **Secant lines** *intersect* the curve at **[1 | 2]** point(s). Slope = **[AVERAGE | INSTANTANEOUS]** rate of change.
  - **Tangent lines** ________ the curve at **[1 | 2]** point(s). Slope = **[AVERAGE | INSTANTANEOUS]** rate of change.
    - The slope of a tangent line is also called the **derivative**.

EXAMPLE Given $f(x) = x^2$, find the slope of the tangent line at $x = 1$.

Recall **Secant (Average RoC)**	New **Tangent (Instantaneous RoC)**
$m_{\text{sec}} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$	$m_{\tan} = \frac{f(x) - f(c)}{x - c}$ $\quad c =$ ___



$$m_{\text{sec}} = \frac{f(3) - f(1)}{3 - 1} = \frac{3^2 - 1^2}{3 - 1} = \frac{9 - 1}{2} = \frac{8}{2} = 4$$



$$m_{\tan} = \frac{f(\quad) - f(1)}{\quad - 1}$$

$$= \lim_{x \to 1}$$

$$= \lim_{x \to 1}$$

$$= \lim_{x \to 1}$$

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PRACTICE Given the function $f(x) = 4x^2 - 1$ calculate the slope of the tangent line at $x = -3$.

PRACTICE Given the function $f(x) = x^2 - 10x + 2$ calculate the slope of the tangent line at $x = 2$.

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PRACTICE Given the function $f(x) = x^2 + 100$ calculate the slope of the tangent line at $x = 0$.

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Equations of Tangent Lines

Recall $m_{\tan} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

- Recall: Tangent lines just *touch* the curve at 1 point.
  - To find eq'n of a tan line, find __________ then plug into point-slope form.

EXAMPLE Find the eq'n of the line tangent to $f(x) = 3x^2 - 4$ at $x = -2$.

x**-val** $(c) =$ ___ y**-val** $(f(c)) =$ ___

$m_{\tan} = \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

HOW TO: Find the Tangent Line to a Curve at a Point

1) Plug x**-val** (__) into $f(x)$ to get y**-val** $(f(c))$ *(if needed)*

2) Plug given $f(x)$, c, and $f(c)$ into

$$m_{\tan} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

3) Evaluate limit (*might have to _______ & cancel*)

4) Plug $m_{\tan}$ & point into

$$(y - y\text{-val}) = m_{\tan}\,(x - x\text{-val})$$

5) Solve equation for y

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PRACTICE Given the function $f(x) = 3(x^2 - 1)$ find the equation of the tangent line at $x = 1$.

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EXAMPLE Find the equation of the line tangent to $f(x) = -4x^2$ at $x = -2$.

$m_{\tan} = \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

$= \lim_{x \to}$

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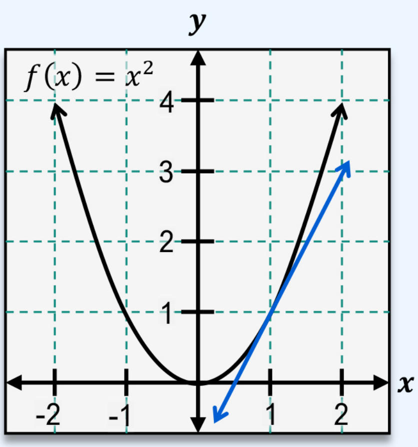
Derivatives

- Recall: **Derivative** is just the slope of the __________ line. Notations: $f'(x),\ y',\ \frac{dy}{dx}$
  (instantaneous rate of change)
  - To write the eq'n for the derivative of a function at *any* point on the curve, use the **Definition of the Derivative**:

EXAMPLE Find the derivative of $f(x) = x^2$ for **any** x. Use this to find the slope of the tangent line at $x = 1$ and $x = -2$.

Recall — Tangent Line	New — Derivatives
$m_{\tan} = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$	$\_\_(x) = \lim_{h\to} \frac{f(\quad\quad) - f(\quad)}{}$



$m_{\tan} =$

$\lim_{x\to 1} \frac{f(x) - f(1)}{x-1}$

$= \lim_{x\to 1} \frac{x^2 - 1^2}{x-1}$

$= \lim_{x\to 1} \frac{(x+1)(x-1)}{x-1}$

$= \lim_{x\to 1} x + 1$

$= 2$

$f(x) = x^2$

$f'(x) = \lim_{h\to 0}$ $\qquad$ $f'(1) =$

$= \lim_{h\to 0}$

$= \lim_{h\to 0}$ $\qquad$ $f'(-2) =$

$= \lim_{h\to 0}$

TOPIC: TANGENT LINES AND DERIVATIVES

PRACTICE Find the derivative of the function $f(x) = 4x^2 - 9x$.

PRACTICE Use the definition of a derivative to find the derivative of the function $g(x) = x^3$ at $x = -1$.