

TOPIC: MULTIPLYING AND DIVIDING COMPLEX NUMBERS

Multiplying Complex Numbers

◆ Complex numbers are multiplied just like algebraic expressions! We A) _____ or B) _____

► Multiplying will *ALWAYS* produce an i^2 term that will get simplified.

EXAMPLE: Find the product. Write answers in standard form.

(A) $3i(7 - 2i)$

(B) $(-6 + 2i)(3 + 4i)$

MULTIPLYING COMPLEX NUMBERS

- 1) Distribute or FOIL
- 2) Apply $i^2 = -1$
- 3) Combine Like Terms

PRACTICE

Perform the indicated operation. Express your answer in standard form.

$$(3 + 8i)^2$$

PRACTICE

Find the product. Express your answer in standard form.

$$2i(9 - 4i)(6 + 5i)$$

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PRACTICE

Multiply the following and simplify.

(A) $(5 - i)(12)$

(B) $(13i)(17i)$

(C) $(7 + 3i)(7 - 3i)$

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Complex Conjugates

- ◆ Reverse the _____ of *only* the **imaginary** part of a complex number to get the **conjugate**: $a + bi \Leftrightarrow$

EXAMPLE: Find the conjugate of each complex number.

(A)

$$1 + 2i$$

(B)

$$1 - 2i$$

(C)

$$-1 + 2i$$

- ◆ Multiply **complex conjugates** (by FOIL) **ALWAYS** results in a _____ number

EXAMPLE: Find the product.

$$(2 + 3i)(2 - 3i)$$

$$(a + bi)(a - bi) = \underline{\hspace{2cm}}$$

PRACTICE

Find the product of the given complex number and its conjugate.

(A)

$$4 - 5i$$

(B)

$$-7 - i$$

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Dividing Complex Numbers

◆ Dividing by a complex number results in a fraction with i in the bottom: this is **BAD**

► Denominators should **ALWAYS** be real! To do this, multiply by its _____

$$\frac{c}{a + \cancel{bi}}$$

EXAMPLE: Find the quotient. Write answer in standard form.

$$\frac{3}{1 + 2i}$$

DIVIDING COMPLEX NUMBERS

- 1) Multiply **top** AND **bottom** by complex conj. of **bottom** & simplify
- 2) Expand fraction into real & imaginary parts
- 3) Simplify fractions into lowest terms

PRACTICE

Find the quotient. Express your answer in standard form.

(A)

$$\frac{6 + i}{4 - 2i}$$

(B)

$$\frac{-5 + 3i}{-7 - 4i}$$

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Powers of i

- ◆ Recall: $i = \sqrt{-1}$. Many problems will have i raised to the 2nd, 3rd, or even much higher powers!
- All properties of exponents can be applied to powers of i

POWERS OF i	
$\rule{1.5cm}{0.4pt} \rightarrow \rule{1.5cm}{0.4pt}$	$i^5 = \rule{1cm}{0.4pt} = \rule{1cm}{0.4pt} = \rule{1cm}{0.4pt}$
$\rule{1cm}{0.4pt} \rightarrow \rule{1cm}{0.4pt} = \rule{1cm}{0.4pt}$	$i^6 = \rule{1cm}{0.4pt} = \rule{1cm}{0.4pt} = \rule{1cm}{0.4pt}$
$\rule{1cm}{0.4pt} = \rule{1cm}{0.4pt} = \rule{1cm}{0.4pt}$	$i^7 = \rule{2cm}{0.4pt} \rightarrow \rule{1cm}{0.4pt}$
$\rule{1cm}{0.4pt} = \rule{1cm}{0.4pt} = \rule{1cm}{0.4pt}$	$i^8 = \rule{2cm}{0.4pt} \rightarrow \rule{1cm}{0.4pt}$

- **Any** power of i can **ALWAYS** be simplified to $\rule{1cm}{0.4pt}$, $\rule{1cm}{0.4pt}$, $\rule{1cm}{0.4pt}$ or $\rule{1cm}{0.4pt}$

PRACTICE

Evaluate the following powers of i .

(A)

$(3i)^4$

(B)

$(4i)^{-3}$

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How to Evaluate Higher Powers of i

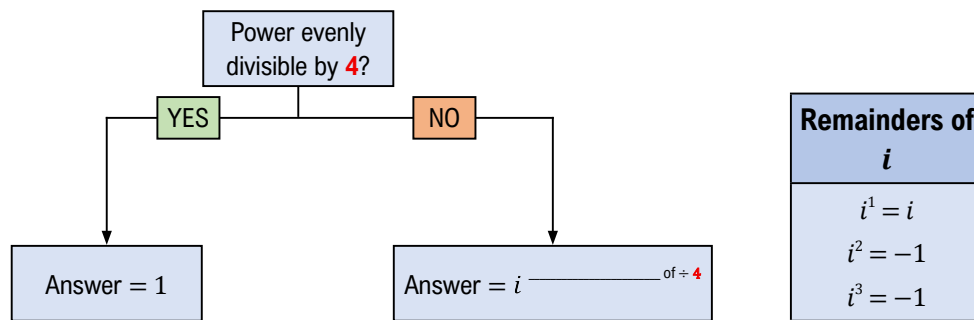
◆ We can express powers of i in terms of _____.

EXAMPLE: Simplify the power of i .

$$\begin{aligned} (A) \quad i^{20} &= i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4 \\ &= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \\ &= \end{aligned}$$

$$\begin{aligned} (B) \quad i^{22} &= \underbrace{i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4}_{i^{20}} \cdot i^2 \\ &= \\ &= \end{aligned}$$

◆ To evaluate i raised to a *very high* power, here's a shortcut:



EXAMPLE: Simplify the power of i .

$$(A) \quad i^{100}$$

$$(B) \quad i^{22}$$

$$(C) \quad i^{67}$$

PRACTICE

Simplify the power of i .

$$(A) \quad i^{1003}$$

$$(B) \quad i^{85}$$

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EXAMPLE

Perform the indicated operation and simplify where possible.

(A) $i^{12} + i^{15}$

(B) $i^8 \times i^5$

(C) $\frac{i^{10} + i^{15}}{i^9 - i^{12}}$