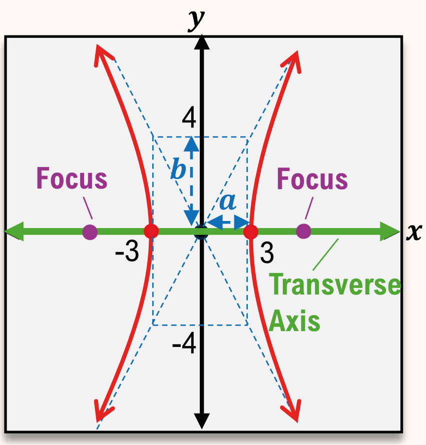
TOPIC: HYPERBOLAS

Intro to Hyperbolas

◆ Hyperbolas have two curved **branches** guided by ________________ & **vertices** that lie on a **transverse axis**.

► For all the points on the branches, the *difference* in distance between 2 fixed points (**foci**) is constant.

Horizontal Hyperbola

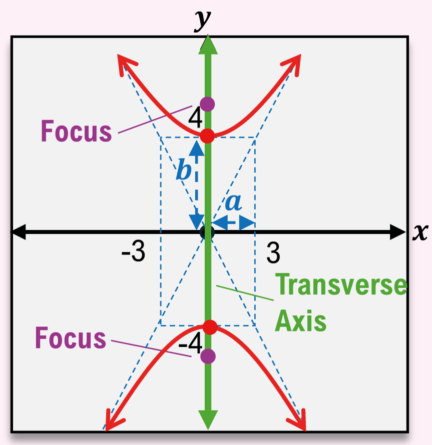


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Fundamental Rectangle:

Where $a =$ ___ & $b =$ ___

Vertical Hyperbola



$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

Fundamental Rectangle:

Where $a =$ ___ & $b =$ ___

◆ The eqn of a hyperbola is like an ellipse except w/ $-$ instead of $+$ & the _______ of x^2 & y^2 matters.

Recall

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(Ellipse)

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PRACTICE Determine if the transverse axis is horizontal or vertical for the following hyperbolas.

(*A*) $$\frac{x^2}{12} - \frac{y^2}{16} = 1$$

(*B*) $$\frac{y^2}{9} - \frac{x^2}{9} = 1$$

(*C*) $$3x^2 - y^2 = 18$$

PRACTICE Identify whether the equation is of an ellipse or hyperbola.

(*A*) $$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

(*B*) $$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

(*C*) $$y^2 - \frac{x^2}{16} = 1$$

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Graphing Hyperbolas

◆ Graph hyperbola by starting at center & going __ units left/right & __ units up/down to form **fundamental rectangle**.

► Then, sketch **asymptotes** thru ________ of **fund. rect.**, identify **vertices** & sketch **branches** on **transverse axis**.

New | **Graphing Horizontal Hyperbolas**

Recall

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Center: $(0, 0)$

New

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

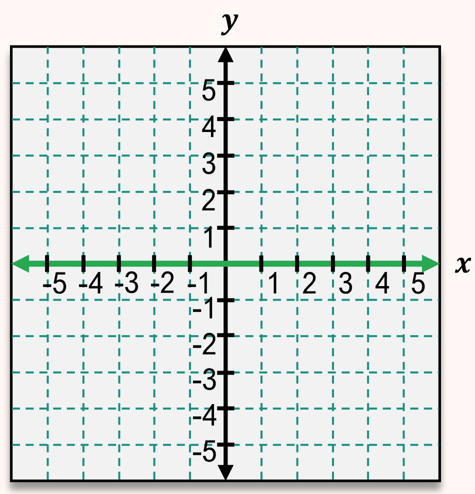
Center: (__, __)

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

Center:

$a =$

$b =$



◆ Use the same strategy to graph vertical hyperbolas:

Recall

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

(Vert. Hyper. at Origin)

New

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

(Vert. Hyper. Not at Origin)

EXAMPLE

Given the equations below, determine the center of the hyperbola and whether it is a vertical or horizontal hyperbola, then sketch a graph.

Recall

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

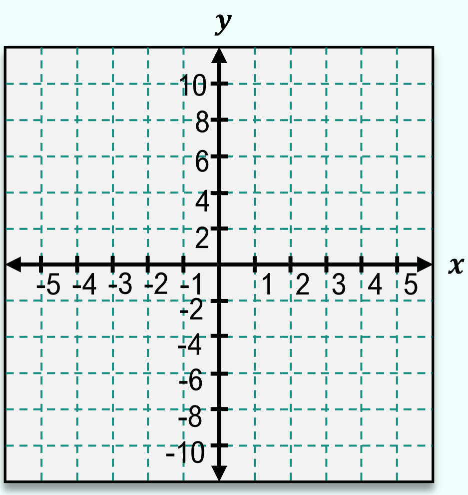
(A) $\frac{x^2}{16} - \frac{y^2}{25} = 1$

Center:

$a =$

$b =$

[HORIZONTAL | VERTICAL]



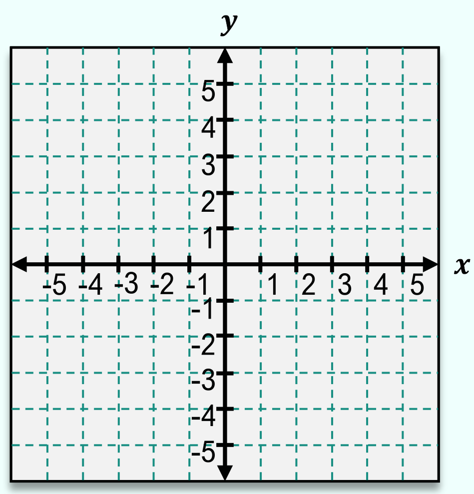
(B) $\frac{y^2}{9} - x^2 = 1$

Center:

$a =$

$b =$

[HORIZONTAL | VERTICAL]



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EXAMPLE Given the equations below, determine the center of the hyperbola and whether it is a vertical or horizontal hyperbola, then sketch a graph.

Recall

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

$(\boldsymbol{A})$

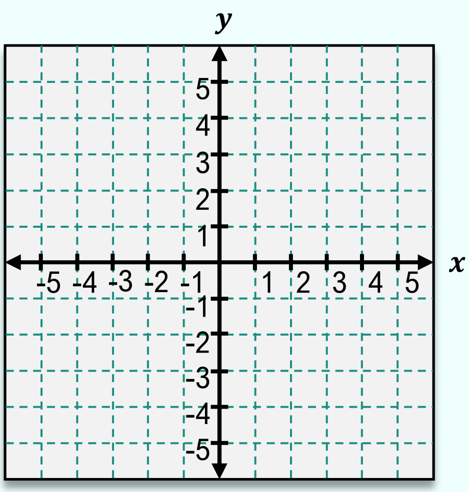
$$\frac{(x-2)^2}{4} - \frac{(y-1)^2}{9} = 1$$

Center:

$\boldsymbol{a} =$

$\boldsymbol{b} =$

[HORIZONTAL | VERTICAL]



$(\boldsymbol{B})$

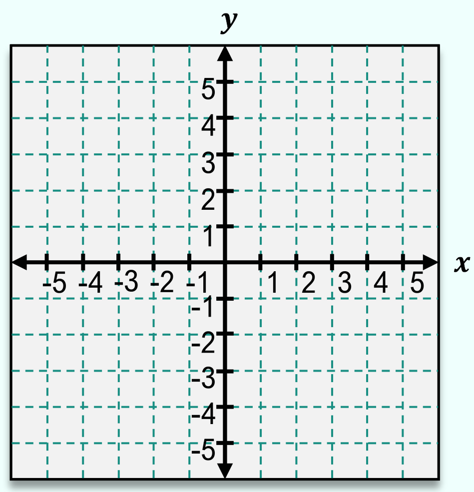
$$\frac{y^2}{16} - (x+2)^2 = 1$$

Center:

$\boldsymbol{a} =$

$\boldsymbol{b} =$

[HORIZONTAL | VERTICAL]



PRACTICE Identify whether the graph of each equation is a circle, an ellipse, a hyperbola, or a parabola.

$(\boldsymbol{A})$ $5x^2 + 5y^2 = 180$

$(\boldsymbol{B})$ $\dfrac{x^2}{100} + y^2 = 1$

$(\boldsymbol{C})$ $4x^2 - 9y^2 = 36$

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EXAMPLE Graph the given vertical hyperbola. Compare it with the given horizontal hyperbola.

Recall

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(Horiz. Hyper at Origin)

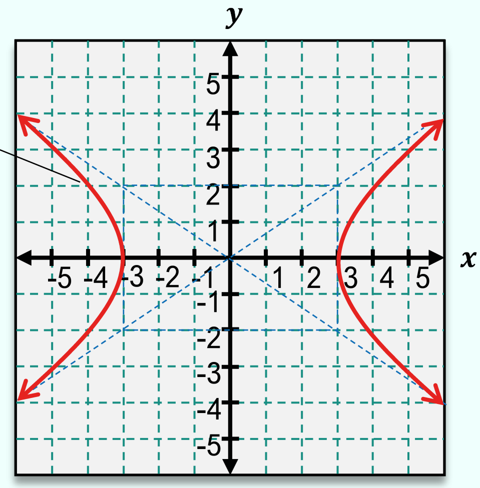
Recall

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

(Vert. Hyper at Origin)

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$



Center:

$\boldsymbol{a} =$

$\boldsymbol{b} =$