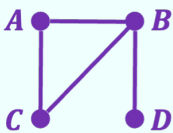
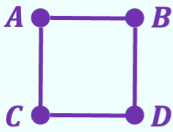
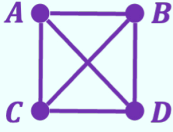


TOPIC: EULER'S THEOREMS

Euler's Theorems

♦ Recall: An Euler path travels along each edge in a graph *exactly* once.

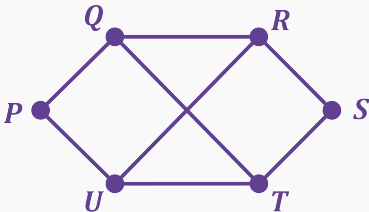
► **Euler's Theorems** use the number of _____ vertices in a graph to determine if an Euler path or circuit exists.

If the graph has...	Then there is...		Example
___ odd vertices	At least 1 Euler [PATH CIRCUIT], but NO Euler [PATH CIRCUIT]	Starts at _____ <i>odd</i> vertex Ends at _____ <i>odd</i> vertex	
___ odd vertices	At least 1 Euler [PATH CIRCUIT]	Starts & ends at _____ vertex	
___ odd vertices	___ Euler path or circuit		

PRACTICE

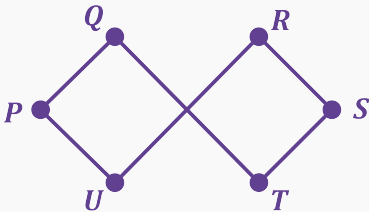
Determine if the graph has an Euler Path (E.P.), an Euler Circuit (E.C.) or neither.

(A)



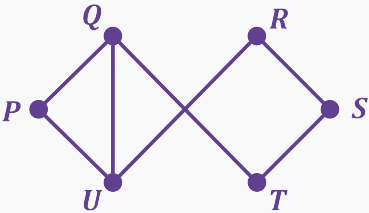
[E.P. | E.C. | NEITHER]

(B)



[E.P. | E.C. | NEITHER]

(C)



[E.P. | E.C. | NEITHER]

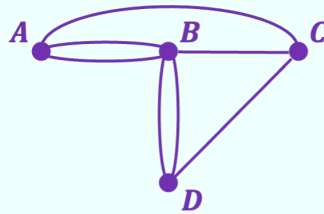
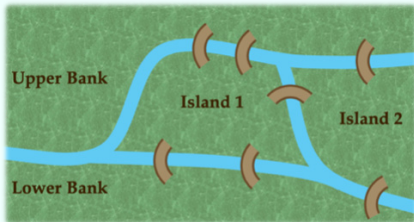
Euler's Theorems

- 2 odd vert. ⇒ Euler path
- 0 odd vert. ⇒ Euler circuit
- > 2 odd vert. ⇒ Neither

TOPIC: EULER'S THEOREMS

EXAMPLE

The graph below represents a map of Königsberg, where vertices are land masses and edges are the bridges that connect them. Is there a way to cross each bridge exactly once? Explain.



Euler's Theorems



2 odd vert. $\Rightarrow$ Euler path

0 odd vert. $\Rightarrow$ Euler circuit

> 2 odd vert. $\Rightarrow$ Neither

TOPIC: EULER'S THEOREMS

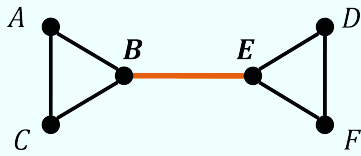
Fleury's Algorithm (Burning Bridges Rule)

◆ **Fleury's Algorithm** is a way to find an Euler path or circuit, by avoiding **bridges**.

► A **bridge** is an edge that connects 2 otherwise disconnected sections of a graph.

EXAMPLE

Use Fleury's algorithm to find an Euler path on the graph.



Euler's Theorems

- 2 odd vert. $\Rightarrow$ Euler path
- 0 odd vert. $\Rightarrow$ Euler circuit
- > 2 odd vert. $\Rightarrow$ Neither

Vertices:

Edges:

HOW TO: Find Euler Path or Circuit

- 1) Determine if EP or EC exists using ET
- 2) If 2 odd vert., start at ____ vertex
If 0 odd vert., start at ____ vertex
- 3) _____ one edge at a time
 - After edge is traced, mark it off
 - Only trace a **bridge** if it's the ____ option

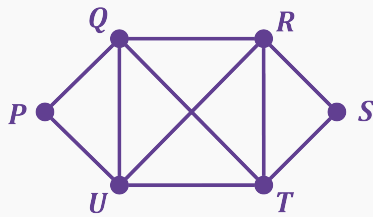
"Don't burn your bridges!"

TOPIC: EULER'S THEOREMS

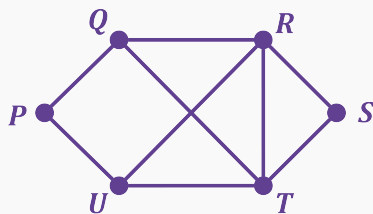
PRACTICE

Use Fleury's alg. to find an Euler path or circuit.

(A)



(B)



HOW TO: Find Euler Path or Circuit

- 1) Determine if EP or EC exists
- 2) If 2 odd vert., start at odd vert.
If 0 odd vert., start at any vert.
- 3) Trace one edge at a time
 - After edge is traced, mark it off
 - Only trace a bridge if it's the last option

EXAMPLE

A mail carrier delivers to the neighborhood pictured.

(A) Draw a graph to represent the neighborhood.



(Hint: Use street blocks = edges,
intersections & corners = vertices)



(B) Does an Euler path or circuit exist in the graph?

(C) The mail carrier must cover each street block exactly once.

Use Fleury's algorithm to find a route.