TOPIC: PROPERTIES OF LOGARITHMS

Product Property of Logs

◆ Properties of *logarithms* come from corresponding properties of *exponents*: **Recall** $b^m \times b^n = b^{m+n}$

PROPERTIES OF LOGARITHMS		
Name	**Property**	**Description**
Product Property	$\log_b(m \times n) = \log_b m \_\_ \log_b n$ **EX.** $\log_2(4 \cdot 6) =$	*Multiply* terms in argument of _____ log ⇔ **[ADD | SUBTRACT]** _____ logs* **must have SAME base*

◆ Properties of logarithms can be used in _____ directions to **expand** or **condense** log expressions.

EXAMPLE Use the product property to rewrite each logarithm.

(A) $\log_5 3x$

(B) $\log_{10} 7 + \log_{10} 9$

(C) $\log_2 x + \log_3 8$

◆ Note: $\log_b(m+n)$ is NOT equal to $\log_b m + \log_b n$ and cannot be further simplified.

$\log_{10}(7+9)$ $\log_{10} 7 + \log_{10} 9$

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EXAMPLE Simplify the log expression.

$\log_{10} 2 + \log_{10} 5$

Recall

$\log_b b^x = x$

$b^{\log_b x} = x$

$\log_b b = 1$

$\log_b 1 = 0$

(Basic Log Properties)

PRACTICE Rewrite the log expression as a sum of multiple logs. Further simplify if possible.

(A) $\log_{10} (5 \cdot 7)$

(B) $\log_4 (4x)$

(C) $\log_3 (10mn)$

PRACTICE Rewrite the sum as a single logarithm. Further simplify if possible.

(A) $\log_3 4 + \log_3 9$

(B) $\log_5 \frac{1}{3} + \log_{12} y$

(C) $\log_{10} x + \log_{10}(x + 2)$

TOPIC: PROPERTIES OF LOGARITHMS

Quotient Property of Logs

◆ The quotient property of *logarithms* come from corresponding quotient property of *exponents*:

Recall

$$\frac{b^m}{b^n} = b^{m-n}$$

Name	Property	Description
Quotient Property	$\log_b \frac{m}{n} = \log_b m \_\_ \log_b n$	*Divide* terms in argument of single log ⟺ **[ADD | SUBTRACT]** multiple logs* **EX.** $\log_3 \frac{5}{6} =$ **must have SAME base*

EXAMPLE Use the quotient property to rewrite each logarithm.

$(\boldsymbol{A})$ $\log_{10}\left(\frac{7}{x}\right)$

$(\boldsymbol{B})$ $\log_5 32 - \log_5 8$

$(\boldsymbol{C})$ $\log_2 x - \log_2 (x+4)$

◆ Note: $\log_b(m-n)$ is NOT equal to $\log_b m - \log_b n$ and cannot be further simplified.

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PRACTICE Rewrite the log expression as a difference of multiple logs. Further simplify if possible.

(A) $\log_2\left(\frac{x}{6}\right)$

(B) $\log_{12}\left(\frac{1}{4}\right)$

(C) $\log_5\left(\frac{125}{x}\right)$

PRACTICE Rewrite the difference as a single logarithm. Further simplify if possible.

(A) $\log_3(x^2+1) - \log_3(4x-2)$

(B) $\log_5 24 - \log_5 8$

(C) $\log_{10}(8x^2) - \log_{10}(2x)$

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Power Property of Logs

Name	Property	Description
Power Property	$\log_b m^n = \_\_\log_b m$	Term to a *power* in argument of log ⇔ _____ log by power
	EX. $\log_3 7^2 =$	

EXAMPLE Use the power property to rewrite each logarithm.

Recall

$\sqrt{a} = a^{1/2}$	$\frac{1}{a^n} = a^{-n}$

(Exp. Rules)

(A) $4 \log_5 x$

(B) $\log_2 \sqrt{5}$

(C) $\log_{10} \frac{1}{x^3}$

EXAMPLE Use the power property to rewrite the log expression.

$\log_{10} \sqrt[3]{y^4}$

PRACTICE Use the power property to rewrite the log expression.

(A) $\log_9 5^4$

(B) $\log_6 \frac{1}{\sqrt{m}}$

(C) $\log_2 (x+1)^2$

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Expanding & Condensing Log Expressions Using Multiple Properties

◆ You can expand or condense more complicated logarithmic expressions by applying *multiple* log properties.

► When condensing logs, always apply the power rule _____.

EXAMPLE

(***A***) Write the expression as the sum or difference of multiple logs.

$\log_{10} 3xy^2$

Recall

$$\log_b(m \times n) = \log_b m + \log_b n$$

$$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$$

$$\log_b m^n = n \log_b m$$

(***B***) Write the expression as a single log.

$4 \log_5 x - \log_5 (x + 2)$

TOPIC: PROPERTIES OF LOGARITHMS

PRACTICE Rewrite the log expression as the sum or difference of multiple logs.

Recall

$$\log_b(m \times n) = \log_b m + \log_b n$$

$$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$$

$$\log_b m^n = n \log_b m$$

(***A***) $\log_{10}\left(\frac{8x^3}{5y}\right)$

(***B***) $\log_2\left(\frac{3a^2b^4}{\sqrt{5c^4}}\right)$

PRACTICE Rewrite the log expression as a single log.

(***A***) $\log_3 5 + 2\log_3 x - \log_3 2$

(***B***) $3\log_3 a - \frac{1}{2}\log_4 (b+1) + \log_4 (4c)$

TOPIC: PROPERTIES OF LOGARITHMS

EXAMPLE Given $\log_x 3 = -1.585$, $\log_x 5 = -2.322$, approximate the following values, to three decimal places.

(A)

$$\log_x \frac{45}{\sqrt{3}}$$

(B)

$$\log_x \left(\frac{75}{3\sqrt{5}}\right)$$

PRACTICE Determine if the given log statement is true or false.

(A)

$$\log_2 48 = \log_2 3 + 4\log_2 2$$

[TRUE | FALSE]

(B)

$$\log_3 \left(\frac{324}{4\sqrt{3}}\right) = 4\log_3 3 + \log_3 3 - \frac{1}{2}\log_3 4$$

[TRUE | FALSE]