TOPIC: MULTIPLYING AND DIVIDING COMPLEX NUMBERS

Multiplying Complex Numbers

◆ Complex numbers are multiplied just like algebraic expressions! We A) _____ or B) _____

► Multiplying will *ALWAYS* produce an i^2 term that will get simplified.

EXAMPLE: Find the product. Write answers in standard form.

(A) $3i(7-2i)$

(B) $(-6+2i)(3+4i)$

MULTIPLYING COMPLEX NUMBERS

1) Distribute or FOIL

2) Apply $i^2 = -1$

3) Combine Like Terms

PRACTICE Perform the indicated operation. Express your answer in standard form.

$(3+8i)^2$

PRACTICE Find the product. Express your answer in standard form.

$2i(9-4i)(6+5i)$

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PRACTICE Multiply the following and simplify.

$(\boldsymbol{A})$ $(5 - i)(12)$

$(\boldsymbol{B})$ $(13i)(17i)$

$(\boldsymbol{C})$ $(7 + 3i)(7 - 3i)$

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Complex Conjugates

◆ Reverse the _____ of *only* the **imaginary** part of a complex number to get the **conjugate:** $a + bi \Leftrightarrow$

EXAMPLE: Find the conjugate of each complex number.

(A) $1 + 2i$

(B) $1 - 2i$

(C) $-1 + 2i$

◆ Multiplying **complex conjugates** (by FOIL) ***ALWAYS*** results in a _____ number

EXAMPLE: Find the product.

$$(2 + 3i)(2 - 3i)$$

$(a + bi)(a - bi) =$ _____

PRACTICE Find the product of the given complex number and its conjugate.

(A) $4 - 5i$

(B) $-7 - i$

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Dividing Complex Numbers

$$\frac{c}{a+bi}$$

◆ Dividing by a complex number results in a fraction with i in the bottom: this is **BAD**

► Denominators should ***ALWAYS*** be real! To do this, multiply by its _____

EXAMPLE: Find the quotient. Write answer in standard form.

$$\frac{3}{1+2i}$$

DIVIDING COMPLEX NUMBERS

1) Multiply top AND bottom by complex conj. of bottom & simplify

2) Expand fraction into real & imaginary parts

3) Simplify fractions into lowest terms

PRACTICE Find the quotient. Express your answer in standard form.

(A)

$$\frac{6+i}{4-2i}$$

(B)

$$\frac{-5+3i}{-7-4i}$$

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Powers of i

◆ Recall: $i = \sqrt{-1}$. Many problems will have i raised to the 2nd, 3rd, or even much higher powers!

► All properties of exponents can be applied to powers of i

POWERS OF i	
$i^1 =$ ⟶ ____	$i^5 =$ _____ $=$ _____ $=$ ____
$i^2 =$ ⟶ _____ $=$ ____	$i^6 =$ _____ $=$ _____ $=$ ____
$i^3 =$ _____ $=$ _____ $=$ ____	$i^7 =$ ⟶ ____
$i^4 =$ _____ $=$ _____ $=$ ____	$i^8 =$ ⟶ ____

► ***Any*** power of i can ***ALWAYS*** be simplified to ___, ___, ___ or ___

PRACTICE Evaluate the following powers of i.

(A) $(3i)^4$

(B) $(4i)^{-3}$

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How to Evaluate Higher Powers of i

◆ We can express powers of i in terms of _____.

EXAMPLE: Simplify the power of i.

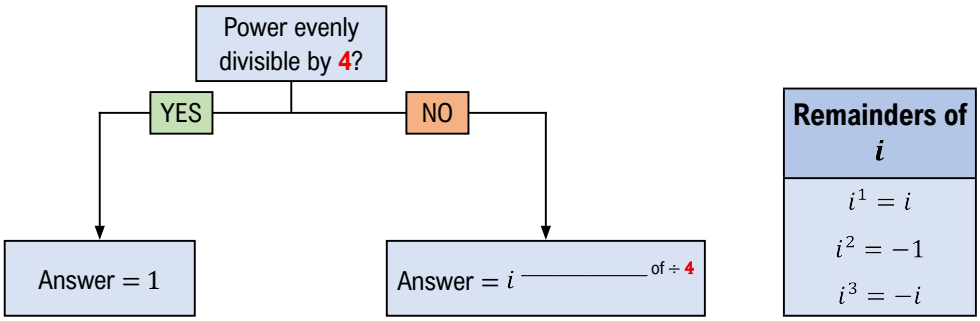
(A)

$$i^{20} = i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4$$
$$= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$$
$$=$$

(B)

$$i^{22} = \underbrace{i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4} \cdot i^2$$
$$=$$
$$=$$

◆ To evaluate i raised to a *very high* power, here's a shortcut:



EXAMPLE: Simplify the power of i.

(A) i^{100}

(B) i^{22}

(C) i^{67}

PRACTICE Simpllify the power of i.

(A) i^{1003}

(B) i^{85}

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EXAMPLE Perform the indicated operation and simplify where possible.

(A)

$$i^{12} + i^{15}$$

(B)

$$i^{8} \times i^{5}$$

(C)

$$\frac{i^{10} + i^{15}}{i^{9} - i^{12}}$$