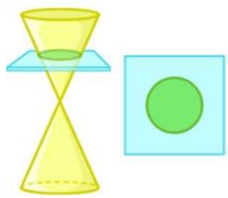
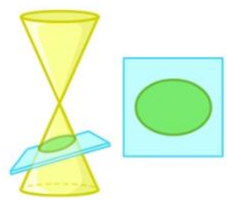
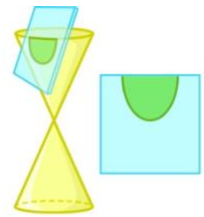
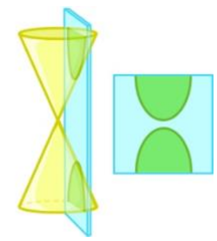
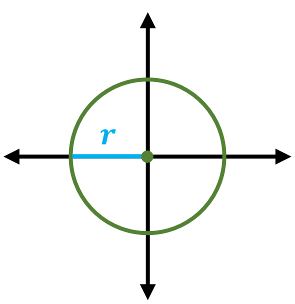
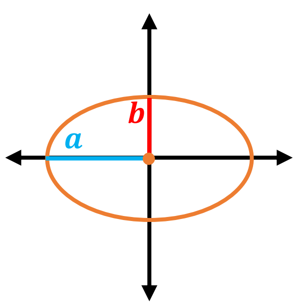
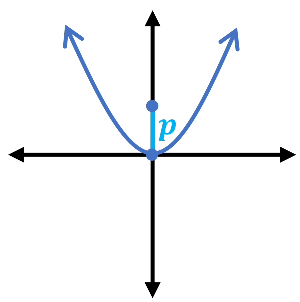
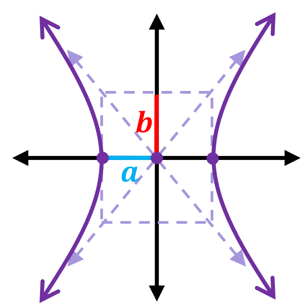
TOPIC: CONIC SECTIONS
Geometries from Conic Sections

- You will need to *graph*, write *equations* for, & identify *characteristics* of the following shapes
  - These shapes (**Conic Sections**) can be formed by slicing a 3D cone with a 2D plane

Circle	__________	**Parabola**	___________
			
Plane is ___________	Plane is Slightly Tilted	Plane is Heavily Tilted	Plane is __________
			
$x^2 + y^2 = r^2$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y = 4px^2$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

PRACTICE: How can you slice a vertically oriented 3D cone with a 2D plane to get a parabola?

$(\boldsymbol{A})$ Slice the cone with a horizontal plane.
$(\boldsymbol{B})$ Slice the cone with a slightly tilted plane.
$(\boldsymbol{C})$ Slice the cone with a heavily tilted plane.
$(\boldsymbol{D})$ Slice the cone with a vertical plane.

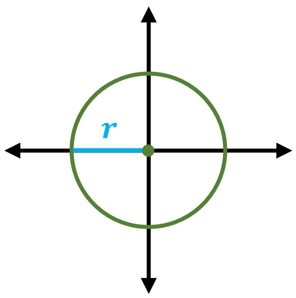
PRACTICE: How can you slice a vertically oriented 3D cone with a 2D plane to get a circle?

$(\boldsymbol{A})$ Slice the cone with a horizontal plane.
$(\boldsymbol{B})$ Slice the cone with a slightly tilted plane.
$(\boldsymbol{C})$ Slice the cone with a heavily tilted plane.
$(\boldsymbol{D})$ Slice the cone with a vertical plane.

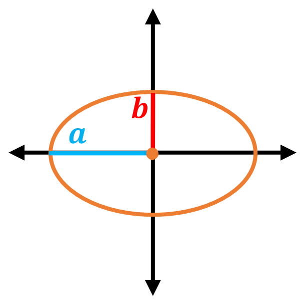
TOPIC: CONIC SECTIONS
Geometries from Conic Sections

PRACTICE**:** A vertically oriented 3D cone is sliced with a *vertical* 2D plane. What is the conic section that will form?

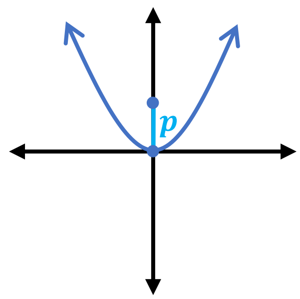
$(\boldsymbol{A})$ Circle



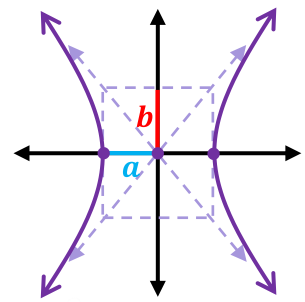
$(\boldsymbol{B})$ Ellipse



$(\boldsymbol{C})$ Parabola

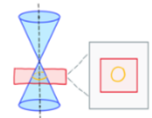


$(\boldsymbol{D})$ Hyperbola

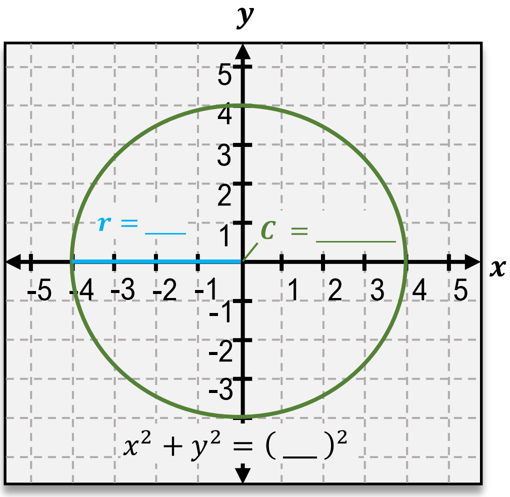
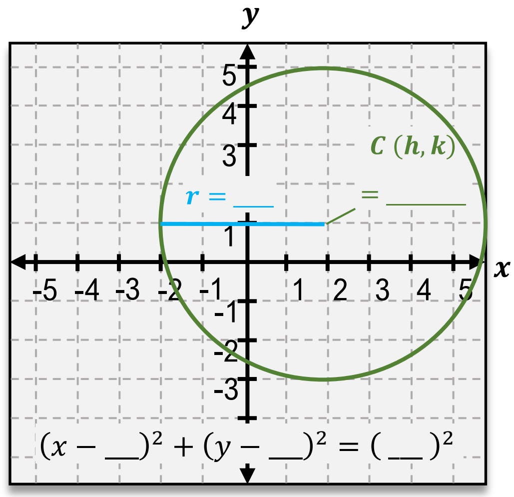


TOPIC: CONIC SECTIONS
Circles in Standard Form

Circle	Ellipse	Parabola	Hyperbola



- You'll need 2 things to graph a circle: __________ (C) & __________ (r)
  - The graph of a circle contains all points which are the _______ distance (r) from the center (C)

Circle at Origin	Circle NOT at Origin
	
$x^2 + y^2 = r^2$	$(x - \mathbf{h})^2 + (y - \mathbf{k})^2 = \mathbf{r}^2$

<u>EXAMPLE</u>: Graph the circle.

$$(x - 1)^2 + (y - 2)^2 = 9$$

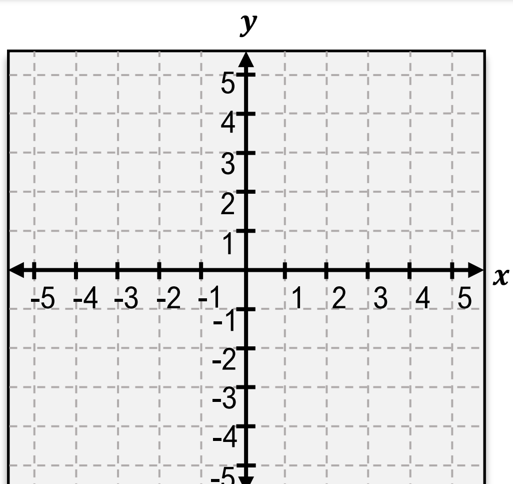
TO GRAPH

1) Center (h, k): (__ , __)

2) Radius: $r =$ _____

3) Plot 4 points a distance $r =$ ____ to the **left**, **right**, **above** and **below** the center point.

4) Connect outside points with a smooth curve

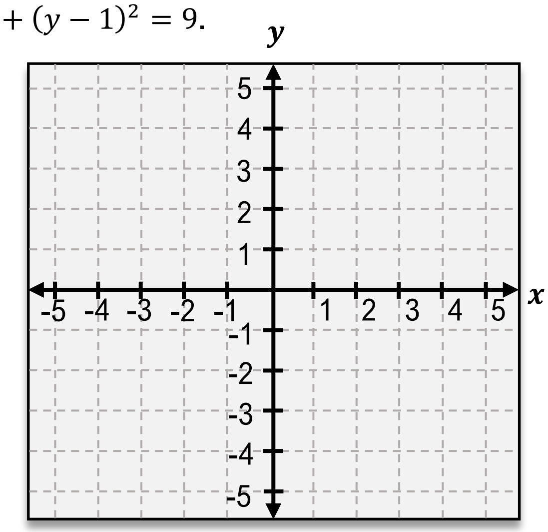


Note: A circle **[<u>IS | IS NOT</u>]** a function because it **[<u>PASSES | FAILS</u>]** the VLT.

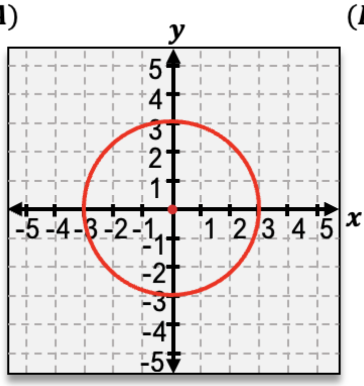
Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

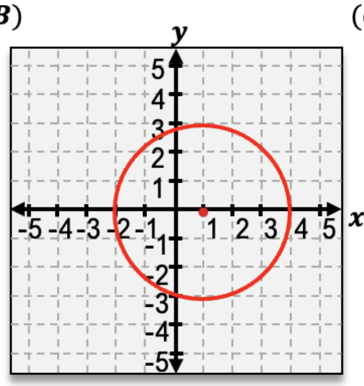
PRACTICE: Sketch a graph of the circle based on the following equation: $x^2 + (y-1)^2 = 9$.



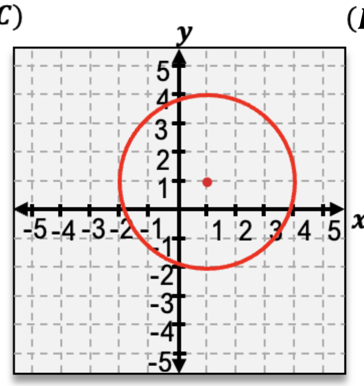
(A)



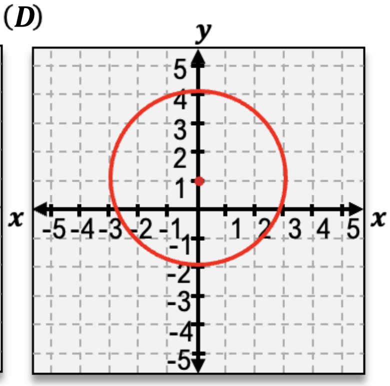
(B)



(C)



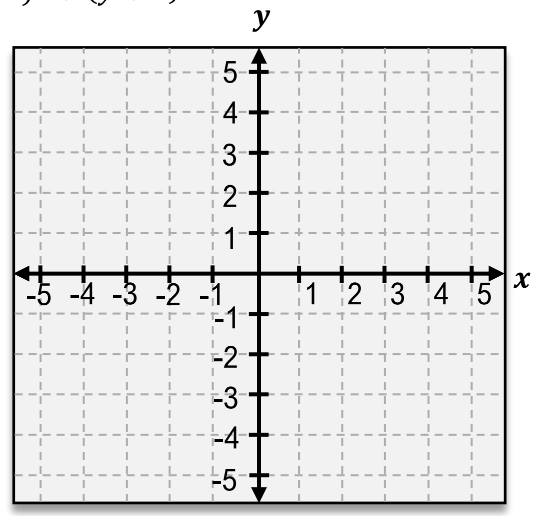
(D)



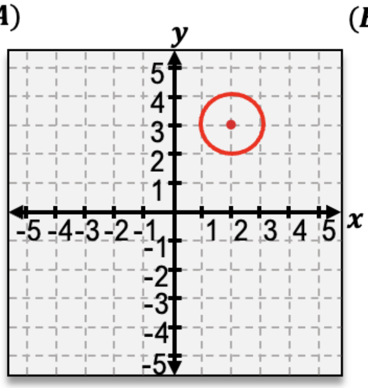
Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

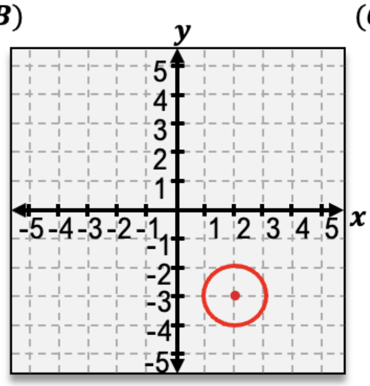
PRACTICE: Sketch a graph of the circle based on the following equation: $(x-2)^2+(y+3)^2=1$.



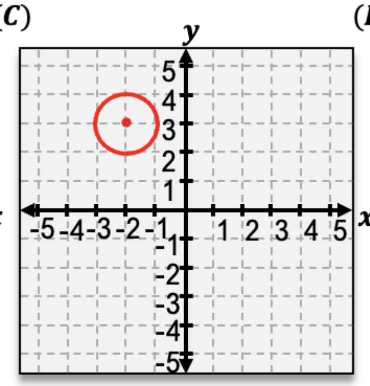
(A)



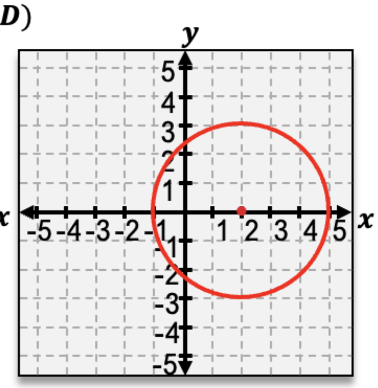
(B)



(C)

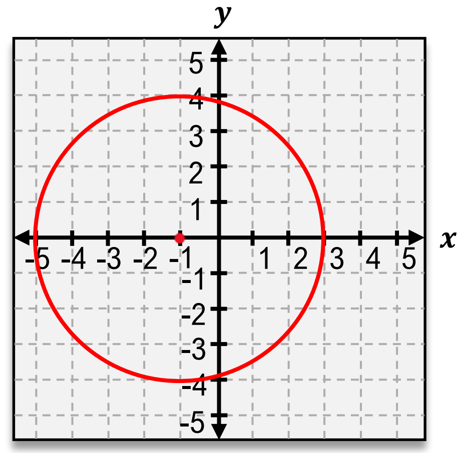


(D)



TOPIC: CONIC SECTIONS

PRACTICE: Find the equation for the following circle.



(A) $x^2 + y^2 = 4$

(B) $(x+1)^2 + y^2 = 4$

(C) $(x-1)^2 + y^2 = 4$

(D) $(x+1)^2 + y^2 = 16$

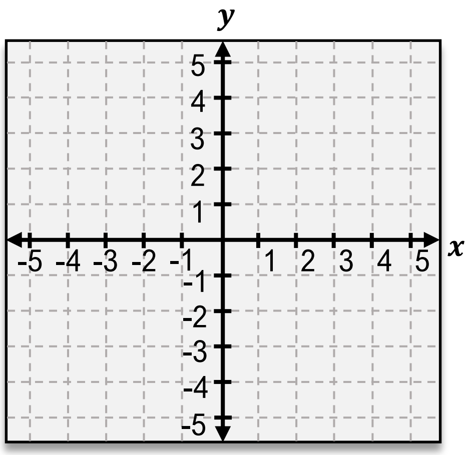
Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

EXAMPLE**:** Write the equation of a circle with the following characteristics and graph it.

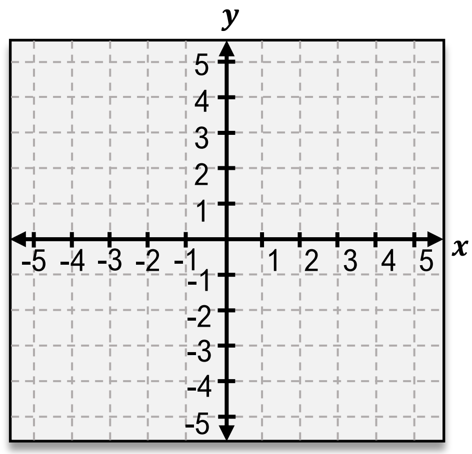
$(\boldsymbol{A})$

Center: $(0,1)$
Radius: 3



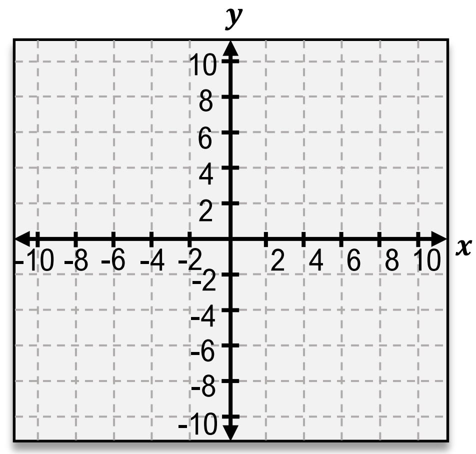
$(\boldsymbol{B})$

Center: $(1,-2)$
Radius: 1



$(\boldsymbol{C})$

Center: $(-6,3)$
Radius: $\sqrt{5}$



TOPIC: CONIC SECTIONS

General Form → Standard Form

Circle	Ellipse	Parabola	Hyperbola

- You will sometimes be given the equation of a circle in **general form.** $x^2+y^2+Ax+By+C=0$
  - Convert to **standard form** by *completing the square* for x & y, then graph.

$(x-h)^2+(y-k)^2=r^2$ Standard Form

$x^2+y^2+2x+6y+8=0$ General Form

↓ Rewrite

$(x^2+2x+\underline{\quad})+(y^2+6y+\underline{\quad})=-8+\underline{\quad}+\underline{\quad}$

↓ Complete the Square

$(x+1)^2+(y+3)^2=2$ Standard Form

EXAMPLE: Convert the following equation to standard form and sketch a graph of the circle.

$x^2+y^2+2x-4y+1=0$

GENERAL FORM → STANDARD FORM CIRCLES

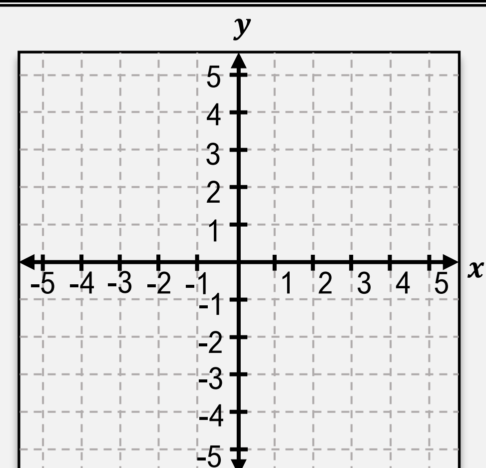
1) Group x terms & y terms on left; constant on right

2) Add _____ to both sides for x terms

Add _____ to both sides for y terms

3) Factor to $\left(x+\underline{\quad}\right)^2$ & simplify

4) Graph from ____________ form



Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

PRACTICE: Determine if the equation $x^2 + y^2 - 2x + 4y - 4 = 0$ is a circle, and if it is, find its center and radius.

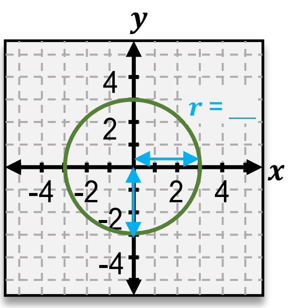
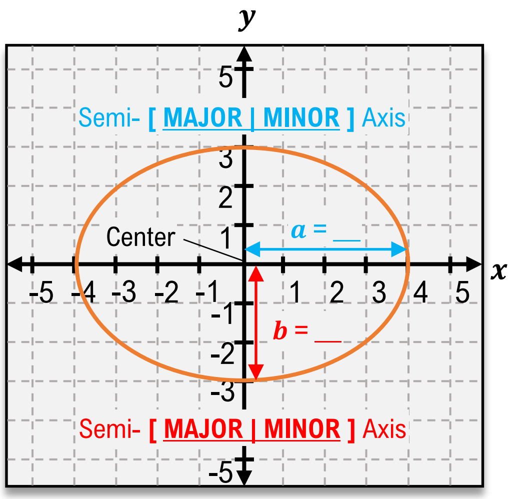
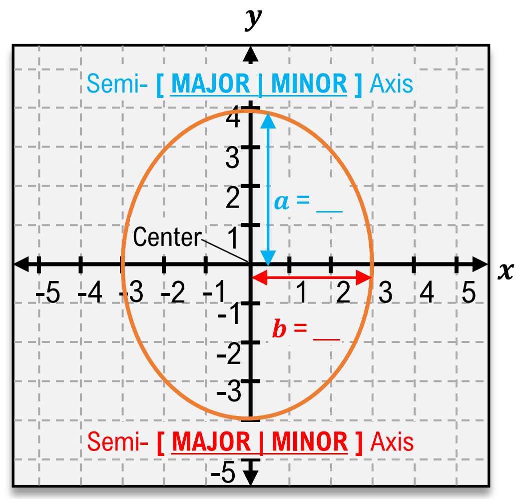
($\boldsymbol{A}$) Is a circle, center $= c(0,0)$, radius $r = 2$.
($\boldsymbol{B}$) Is a circle, center $= c(0,0)$, radius $r = 3$.
($\boldsymbol{C}$) Is a circle, center $= c(1,-2)$, radius $r = 3$.
($\boldsymbol{D}$) Is not a circle.

PRACTICE: Determine if the equation $x^3 + y^2 + 4x - 8y + 4 = 0$ is a circle, and if it is, find its center and radius.

($\boldsymbol{A}$) Is a circle, center $= c(0,0)$, radius $r = 4$.
($\boldsymbol{B}$) Is a circle, center $= c(2,-4)$, radius $r = 4$.
($\boldsymbol{C}$) Is a circle, center $= c(-2,4)$, radius $r = 4$.
($\boldsymbol{D}$) Is not a circle.

TOPIC: CONIC SECTIONS
Graph Ellipses at the Origin

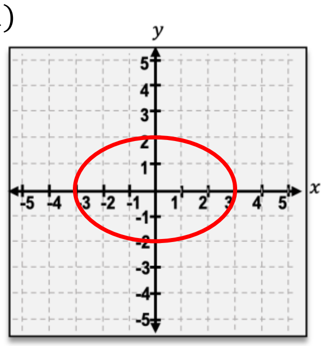
- The equation for a circle depends on its radius.
  - The equation for an ellipse depends on ____ special distances: **semi-major** & **semi-minor axes.**

Circle	Horizontal Ellipse	Vertical Ellipse
		
$x^2 + y^2 = r^2$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{x^2}{\underline{\quad}^2} + \frac{y^2}{\underline{\quad}^2} = 1$

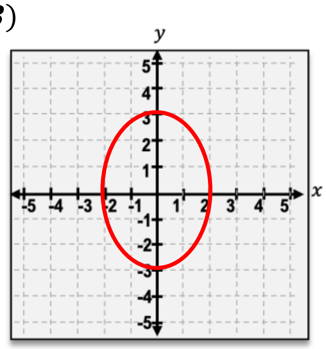
- No matter the orientation, a is always the ____________ distance from the center to the ellipse.

PRACTICE: Given the equation $\frac{x^2}{4} + \frac{y^2}{9} = 1$, sketch a graph of the ellipse.

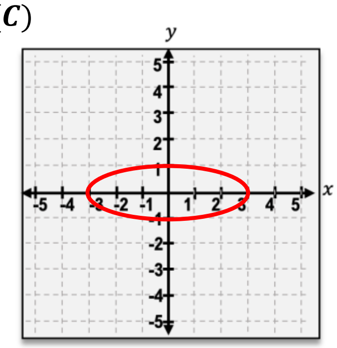
(A)



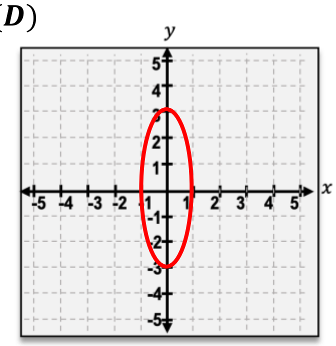
(B)



(C)



(D)



Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

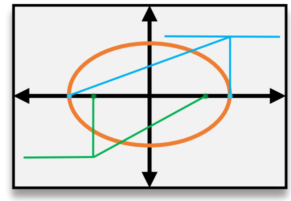
PRACTICE: Given the ellipse equation $\frac{x^2}{16}+\frac{y^2}{4}=1$, determine the magnitude of the semi-major axis (a) and the semi-minor axis (b).

$(\boldsymbol{A})\ a=16, b=4$
$(\boldsymbol{B})\ a=4, b=16$
$(\boldsymbol{C})\ a=4, b=2$
$(\boldsymbol{D})\ a=2, b=4$

Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

Vertices and Foci of Ellipses



- Every ellipse has 2 **Vertices** & 2 **Foci**, both located on the **[MAJOR | MINOR]** axis
  - **Vertices** are the points on the ellipse __________ from the center

    Distance between center & **Vertex** = **[a | b | c]**
  - **Foci** are points inside the ellipse, and the sum of any distance from the **foci** to a single point is ___________

    Distance between center & **Focus** = **[a | b | c]**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$c^2 = a^2 - b^2$$

EXAMPLE: Find the vertices & foci of the ellipse in the graph.

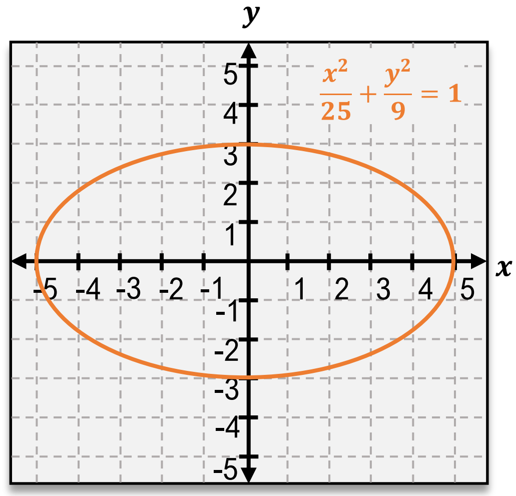
$a =$

$b =$

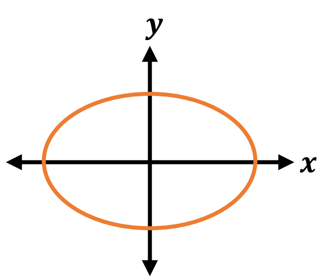
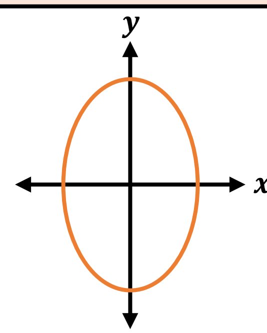
$c =$

Vertices: (__, 0) & (__, 0)

Foci: (__, 0) & (__, 0)



- For a *vertical* ellipse, the coordinates of vertices & foci are different

Horizontal Ellipse	**Vertical Ellipse**
	
Vertices: (a , 0) & ($-a$, 0)	Vertices: (0 , ___) & (0 , ___)
Foci: (c , 0) & ($-c$, 0)	Foci: (0, ___) & (0, ___)
Vertices & Foci on **[x | y]** axis	Vertices & Foci on **[x | y]** axis

<u>TOPIC: CONIC SECTIONS</u>

<u>PRACTICE</u>**:** Determine the vertices and foci of the following ellipse: $\frac{x^2}{49}+\frac{y^2}{36}=1$.

$(\boldsymbol{A})$ Vertices: $(7,0), (-7,0)$
Foci: $(6,0), (-6,0)$
$(\boldsymbol{B})$ Vertices: $(6,0), (-6,0)$
Foci: $(7,0), (-7,0)$
$(\boldsymbol{C})$ Vertices: $(7,0), (-7,0)$
Foci: $\left(\sqrt{13}, 0\right), (-\sqrt{13}, 0)$
$(\boldsymbol{D})$ Vertices: $(0,7), (0,-7)$
Foci: $\left(0, \sqrt{13}\right), (0, -\sqrt{13})$

<u>PRACTICE</u>**:** Determine the vertices and foci of the following ellipse: $\frac{x^2}{9}+\frac{y^2}{16}=1$.

$(\boldsymbol{A})$ Vertices: $(4,0), (-4,0)$
Foci: $\left(\sqrt{7}, 0\right), (-\sqrt{7}, 0)$
$(\boldsymbol{B})$ Vertices: $(0,4), (0,-4)$
Foci: $\left(0, \sqrt{7}\right), (0, -\sqrt{7})$
$(\boldsymbol{C})$ Vertices: $(4,0), (-4,0)$
Foci: $(3,0), (-3,0)$
$(\boldsymbol{D})$ Vertices: $(0,4), (0,-4)$
Foci: $(0,3), (0,-3)$

<u>TOPIC: CONIC SECTIONS</u>

<u>PRACTICE</u>**:** Find the standard form of the equation for an ellipse with the following conditions.

Foci $= (-5,0), (5,0)$

Vertices $= (-8,0), (8,0)$

$(\boldsymbol{A})\ \frac{x^2}{64} + \frac{y^2}{25} = 1$

$(\boldsymbol{B})\ \frac{x^2}{25} + \frac{y^2}{64} = 1$

$(\boldsymbol{C})\ \frac{x^2}{8} + \frac{y^2}{5} = 1$

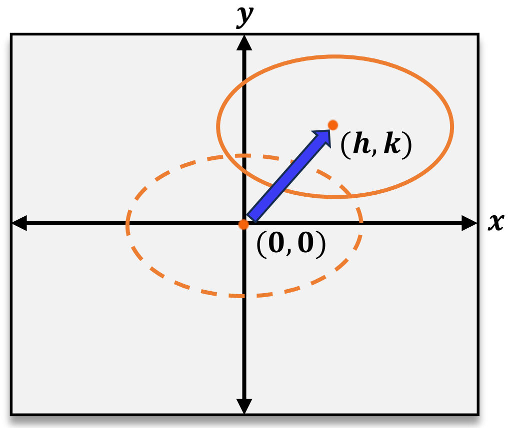
$(\boldsymbol{D})\ \frac{x^2}{64} + \frac{y^2}{39} = 1$

TOPIC: CONIC SECTIONS

Graph Ellipses NOT at the Origin

Circle	Ellipse	Parabola	Hyperbola

- To graph ellipses NOT at the origin, shift points by (h, k)



Horizontal Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

EXAMPLE: Graph the following ellipse.

$$\frac{(x-4)^2}{9} + \frac{(y-2)^2}{64} = 1$$

TO GRAPH

1) Determine the major and minor axis

$a =$ ____ & $b =$ ____

2) Ellipse is **[VERTICAL | HORIZONTAL]**

3) Center (h, k): (__ , __)

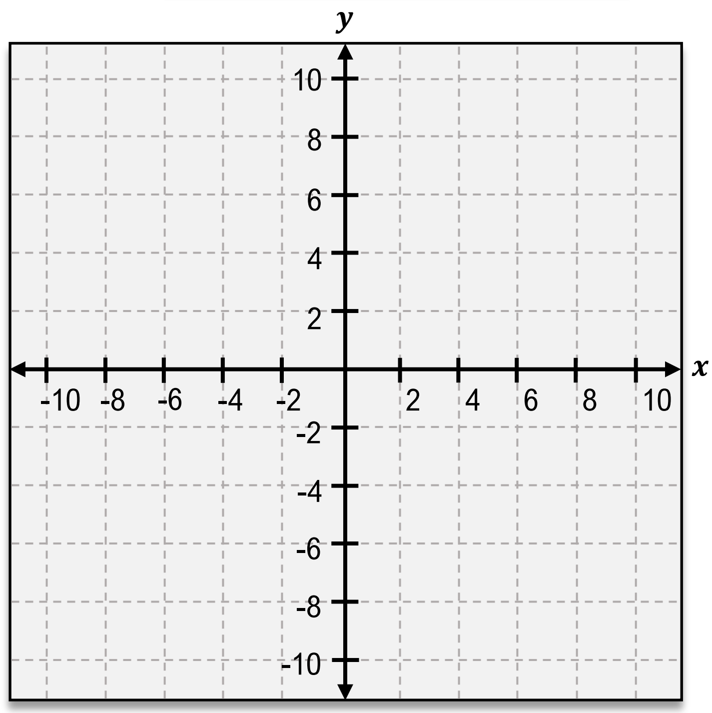
4) Vertices, vert. → $(h, k \pm a)$ *OR* horiz. → $(h \pm a, k)$:

(__ , __) & (__ , __)

5) b points vert. → $(h \pm b, k)$, *OR* horiz. → $(h, k \pm b)$:

(__ , __) & (__ , __)

6) Connect outside points with a smooth curve

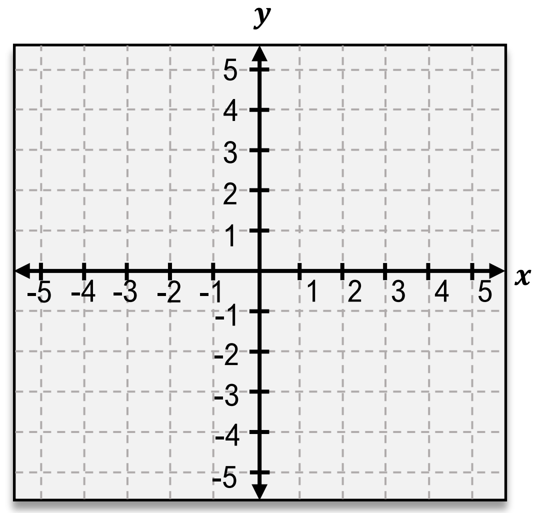


- To find the foci, use $(h, k \pm c)$ for a **[VERT. | HORIZ.]** ellipse, and $(h \pm c, k)$ for a **[VERT. | HORIZ.]** ellipse.

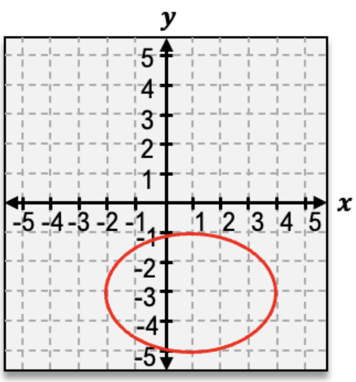
Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

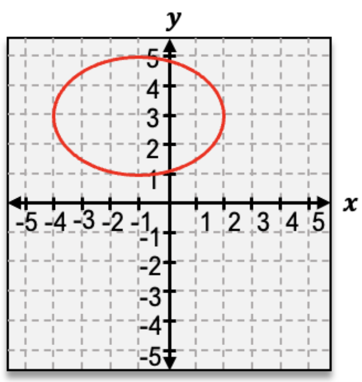
PRACTICE: Graph the ellipse $\frac{(x-1)^2}{9}+\frac{(y+3)^2}{4}=1$.



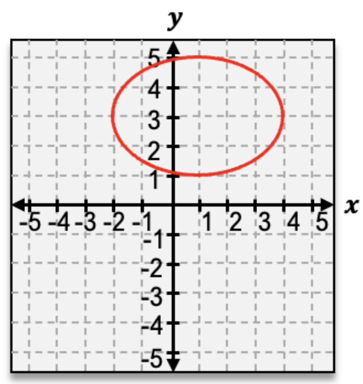
(A)



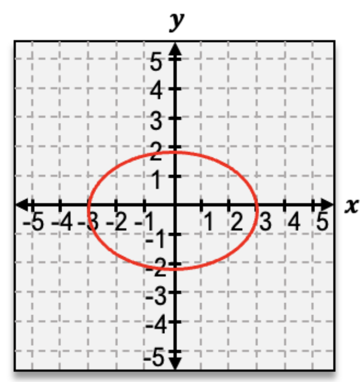
(B)



(C)



(D)



Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

PRACTICE: Determine the vertices and foci of the ellipse $(x+1)^2+\frac{(y-2)^2}{4}=1$.

$(\boldsymbol{A})$ Vertices: $(-1,4),(-1,0)$
Foci: $\left(-1,2+\sqrt{3}\right),(-1,2-\sqrt{3})$

$(\boldsymbol{B})$ Vertices: $(-1,4),(-1,0)$
Foci: $(-2,2),(0,2)$

$(\boldsymbol{C})$ Vertices: $(-2,2),(0,2)$
Foci: $\left(1,2+\sqrt{3}\right),(1,2-\sqrt{3})$

$(\boldsymbol{D})$ Vertices: $(-2,2),(0,2)$
Foci: $\left(2+\sqrt{3},1\right),(2-\sqrt{3},1)$

Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

EXAMPLE: Graph the ellipse and identify the foci.

$$\frac{(x+4)^2}{36}+\frac{(y+3)^2}{49}=1$$

TO GRAPH

1) Ellipse is **[HORIZONTAL | VERTICAL]**

2) Center (h, k): (__ , __)

3) Vertices $(h \pm a, k)$, *OR* $(h, k \pm a)$:

(__ , __) & (__ , __)

4) Points on minor axis $(h \pm b, k)$, *OR* $(h, k \pm b)$:

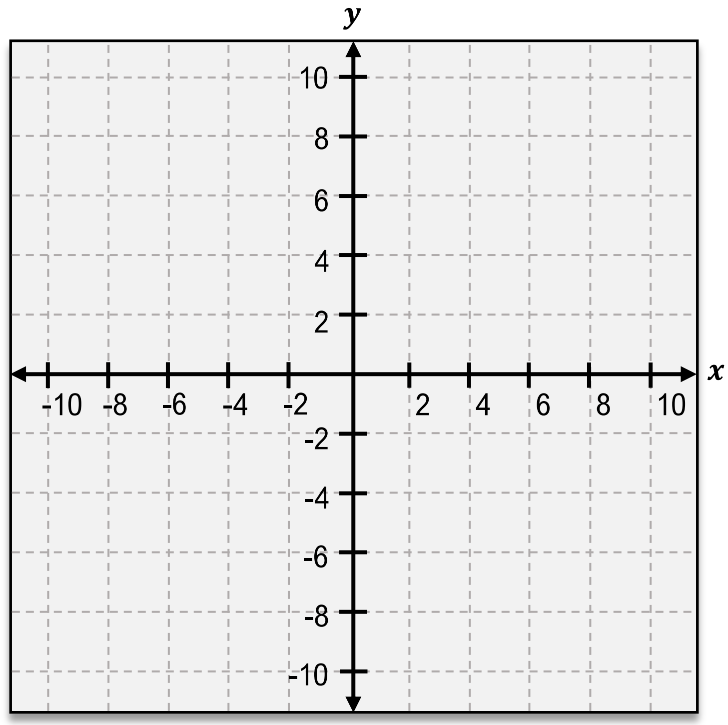
(__ , __) & (__ , __)

5) Connect outer points with a smooth curve

FROM GRAPH

6) Foci $(h \pm c, k)$, *OR* $(h, k \pm c)$:

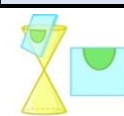
(__ , __) & (__ , __)



TOPIC: CONIC SECTIONS

Parabolas as Conic Sections

Circle	Ellipse	Parabola	Hyperbola



- As conic sections, parabolas have a focus (__________) & directrix (__________)
  - To find **focus**, start at vertex: if opens ↑, go **[↑ | ↓]** $|p|$ units; if opens ↓, go **[↑ | ↓]** $|p|$ units
  - To find **directrix**, start at vertex: if opens ↑, go **[↑ | ↓]** $|p|$ units; if opens ↓, go **[↑ | ↓]** $|p|$ units & draw line

$$y = a(x-h)^2 + k$$

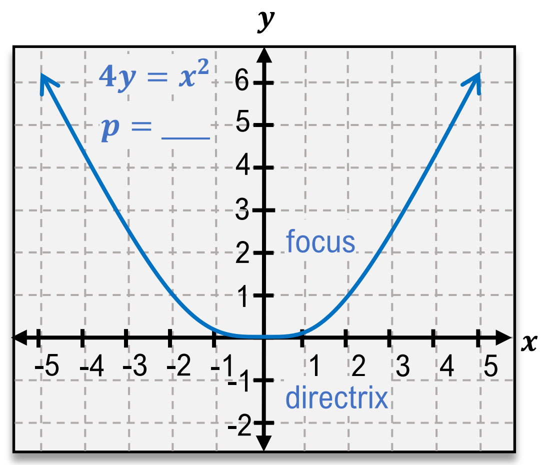
↓

$$4p(y-k) = (x-h)^2$$

↓

$$4py = x^2$$

(Parabola At Origin)



- When p → + the parabola opens **[↑ | ↓]** and when p → − the parabola opens **[↑ | ↓]**

EXAMPLE: Graph the following parabola.

$$\mathbf{8(y-2) = (x-1)^2}$$

TO GRAPH

1) Find the vertex (h, k): (__ , __)

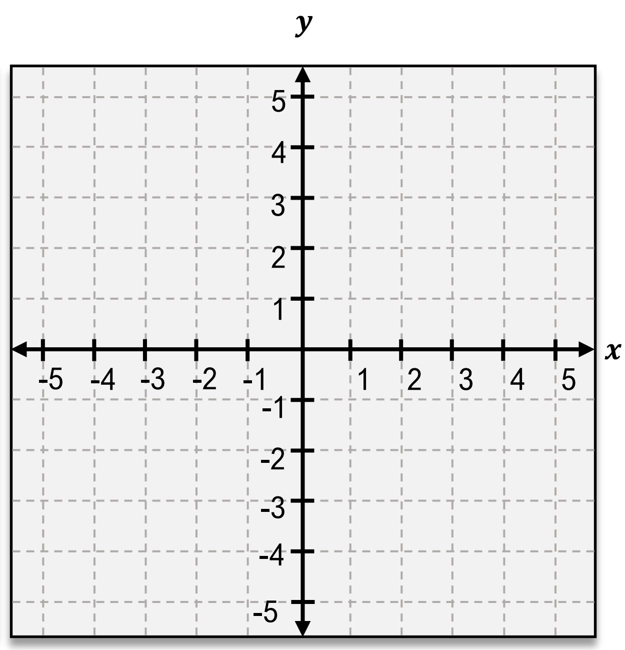
2) Calculate the p value: $p =$ ___

3) Find focus (go **[↑ | ↓]** $|p|$ units from vertex):
(__ , __)

4) From focus, go **left** & **right** $2|p|$ units:
(__ , __) & (__ , __)

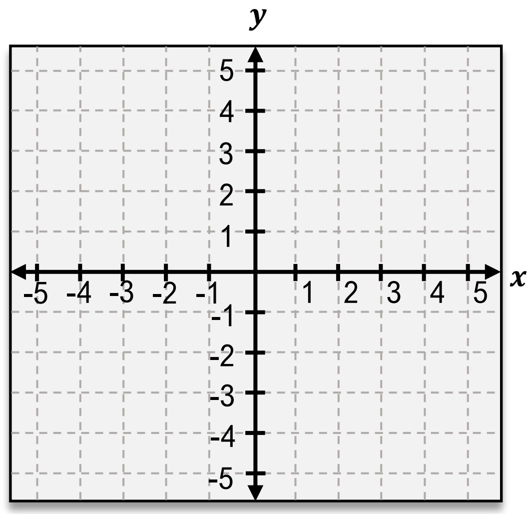
5) Connect outer points with smooth curve

6) Find directrix (go **[↑ | ↓]** $|p|$ units from vertex):
$y =$ ___

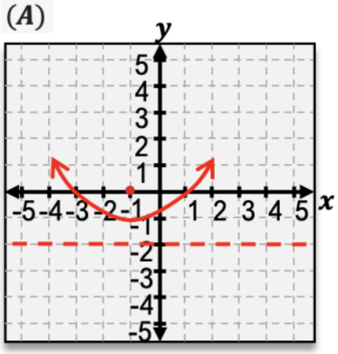


TOPIC: CONIC SECTIONS

PRACTICE: Graph the parabola $-4(y+1)=(x+1)^2$, and find the focus point and directrix line.

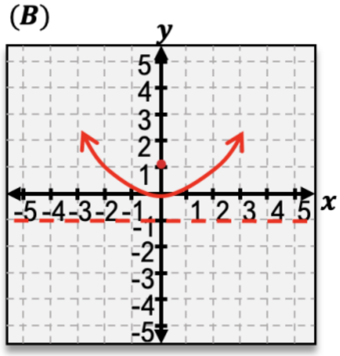


$(\boldsymbol{A})$



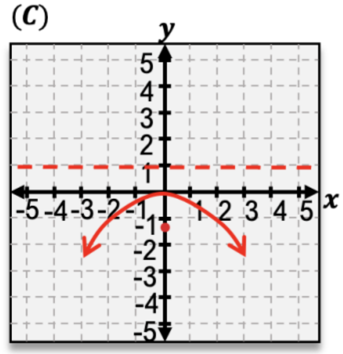
Focus: $(-1,0)$
Directrix: $y=-2$

$(\boldsymbol{B})$



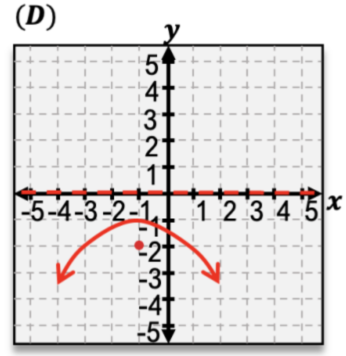
Focus: $(0,1)$
Directrix: $y=-1$

$(\boldsymbol{C})$



Focus: $(0,-1)$
Directrix: $y=1$

$(\boldsymbol{D})$



Focus: $(-1,-2)$
Directrix: $y=0$

PRACTICE: If a parabola has the focus at $(0,-1)$ and a directrix line $y=1$, find the standard equation for the parabola.

$(\boldsymbol{A})$ $4y=x^2$
$(\boldsymbol{B})$ $4(y-1)=x^2$
$(\boldsymbol{C})$ $-4y=x^2$
$(\boldsymbol{D})$ $-4(y+1)=x^2$

Circle	Ellipse	**Parabola**	Hyperbola

TOPIC: CONIC SECTIONS

EXAMPLE**:** Identify the vertex, focus, & directrix of the parabola.

$(\boldsymbol{A})$

$16y = x^2$

$(\boldsymbol{B})$

$\frac{1}{3}y = x^2$

$(\boldsymbol{C})$

$8(y-1) = (x-2)^2$

$(\boldsymbol{D})$

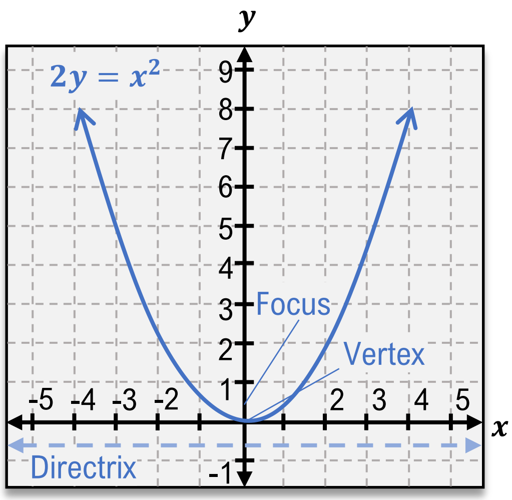
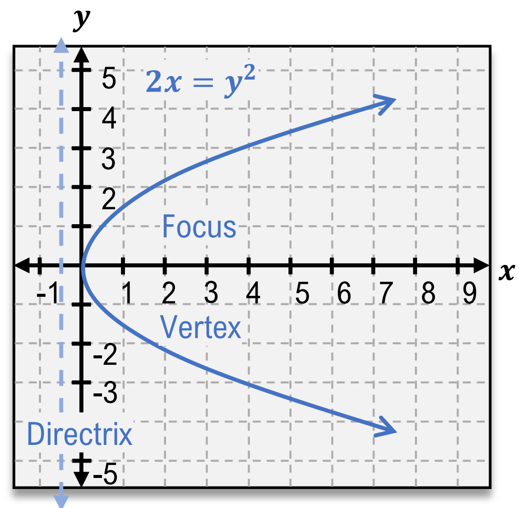
$-12y = (x+1)^2$

TOPIC: CONIC SECTIONS

Horizontal Parabolas

Circle	Ellipse	**Parabola**	Hyperbola

- Horizontal parabolas are just like vertical ones, but with ___ & ___ switched (so they are ___________)
  - The directrix is always ______________ to the axis of symmetry

Vertical Parabola	Horizontal Parabola
	
$4py = x^2$	$4px = y^2$
If p is **pos**: parabola opens **[↑ | ↓ | → | ←]**	If p is **pos**: parabola opens **[↑ | ↓ | → | ←]**
If p is **neg**: parabola opens **[↑ | ↓ | → | ←]**	If p is **neg**: parabola opens **[↑ | ↓ | → | ←]**
Directrix is **[HORIZONTAL | VERTICAL]**	Directrix is **[HORIZONTAL | VERTICAL]**

EXAMPLE**:** Graph the parabola

TO GRAPH

$$8x = y^2$$

1) Find the vertex (h, k): (__ , __)

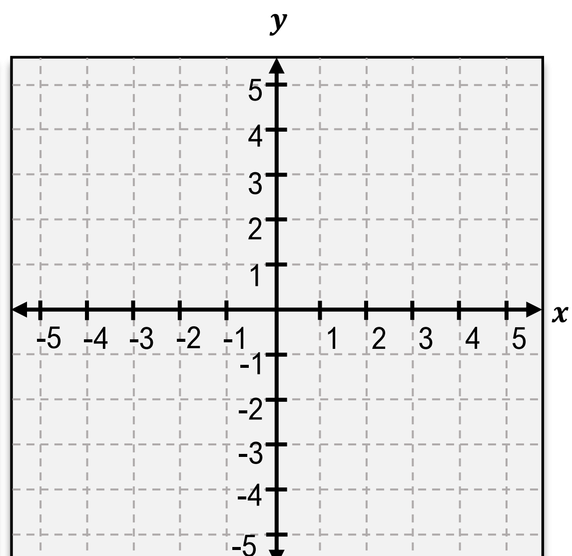
2) Calculate the p value: $|p| =$ ___

3) Find focus (go **[↑ | ↓ |→|←]** $|p|$ units from vertex): (__ , __)

4) From focus, go **[Left & Right | Up & Down]** $|2p|$ units: (__ , __) & (__ , __)

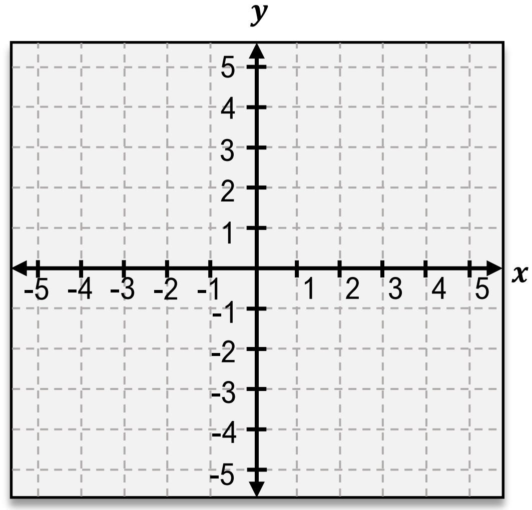
5) Connect outer points with smooth curve

6) Find directrix (go **[↑ | ↓ |→|←]** $|p|$ units from vertex): **[x | y] =** ________

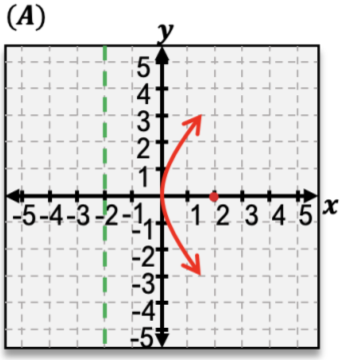


TOPIC: CONIC SECTIONS

PRACTICE: Graph the parabola $8(x+1) = (y-2)^2$, and find the focus point and directrix line.

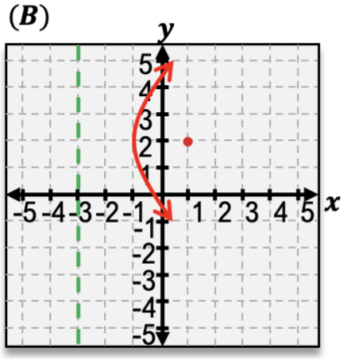


(A)



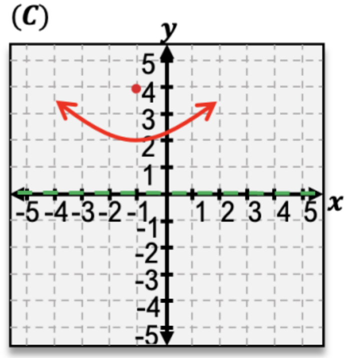
Focus: $(2,0)$
Directrix: $x = -2$

(B)



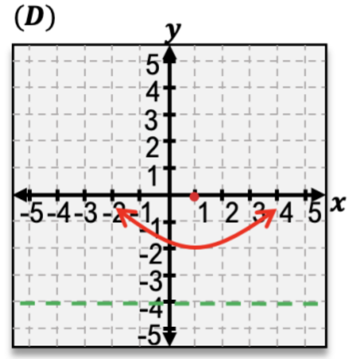
Focus: $(1,2)$
Directrix: $x = -3$

(C)



Focus: $(-1,4)$
Directrix: $y = 0$

(D)



Focus: $(1,0)$
Directrix: $y = -4$

Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

PRACTICE**:** If a parabola has the focus at $(2,4)$ and a directrix line $x = -4$, find the standard equation for the parabola.

$(\boldsymbol{A})$ $12(x+1) = (y-4)^2$
$(\boldsymbol{B})$ $-(x+1) = (y-4)^2$
$(\boldsymbol{C})$ $12x = y^2$
$(\boldsymbol{D})$ $4(x-1) = (y+4)^2$

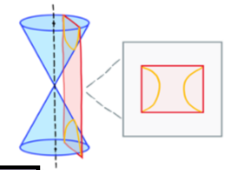
EXAMPLE**:** Identify the vertex, focus, & directrix of the parabola.

$(\boldsymbol{A})$	$(\boldsymbol{B})$	$(\boldsymbol{C})$	$(\boldsymbol{D})$
$4x = y^2$	$9x = (y-2)^2$	$16(x-4) = (y+2)^2$	$-2(x-1) = y^2$

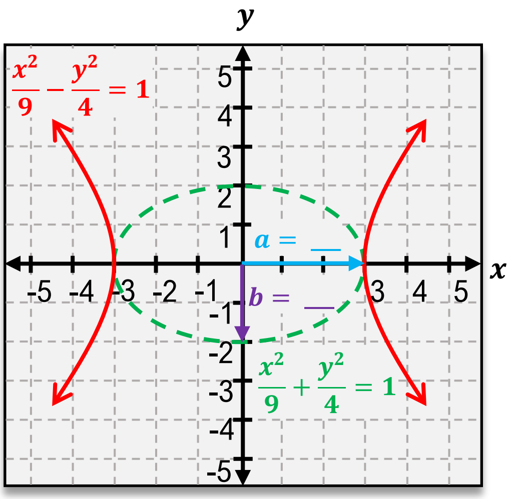
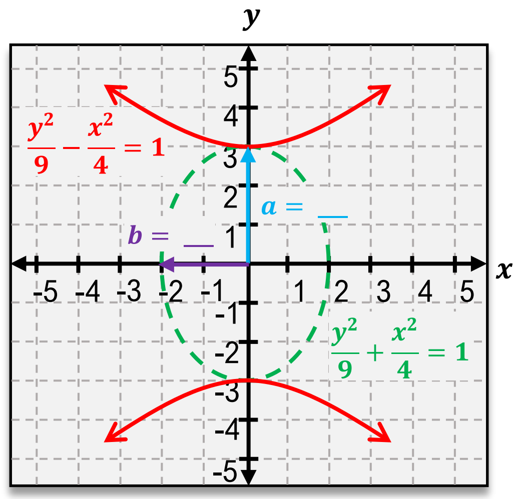
TOPIC: CONIC SECTIONS

Introduction to Hyperbolas

Circle	Ellipse	Parabola	**Hyperbola**



- The equation for a hyperbola is the same as an ellipse, but with a ______.
  - Visually, a hyperbola appears as two ____________ facing away from each other.

Horizontal Hyperbola	**Vertical Hyperbola**
	
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (*Horiz. Ellipse*) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (*Horiz. Hyperbola*)	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ (*Vert. Ellipse*) $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ (*Vert. Hyperbola*)

- The axis major axis (a) is always **[LARGEST | FIRST]** for an ellipse, and **[LARGEST | FIRST]** for a hyperbola.

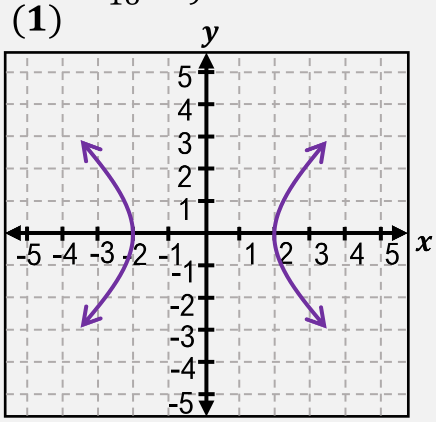
<u>EXAMPLE</u>**:** Match the equation to the graph

$(\boldsymbol{A})$ $\frac{y^2}{16}-\frac{x^2}{9}=1$

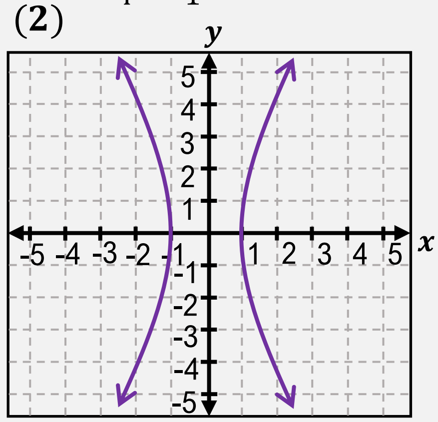
$(\boldsymbol{B})$ $\frac{x^2}{4}-\frac{y^2}{1}=1$

$(\boldsymbol{C})$ $\frac{x^2}{1}-\frac{y^2}{25}=1$

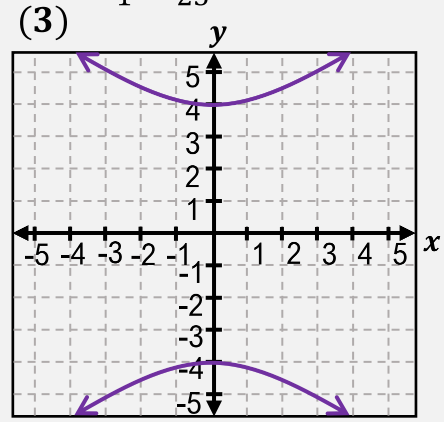
$(\mathbf{1})$



$(\mathbf{2})$



$(\mathbf{3})$



Circle	Ellipse	Parabola	Hyperbola

<u>TOPIC: CONIC SECTIONS</u>

<u>PRACTICE</u>: Given the hyperbola $\frac{x^2}{25}-\frac{y^2}{9}=1$, find the length of the a-axis and b-axis.

$(\boldsymbol{A})\ a=25, b=9$
$(\boldsymbol{B})\ a=9, b=25$
$(\boldsymbol{C})\ a=5, b=3$
$(\boldsymbol{D})\ a=3, b=5$

<u>PRACTICE</u>: Given the hyperbola $x^2-\frac{y^2}{4}=1$, find the length of the a-axis and b-axis.

$(\boldsymbol{A})\ a=1, b=4$
$(\boldsymbol{B})\ a=4, b=1$
$(\boldsymbol{C})\ a=1, b=2$
$(\boldsymbol{D})\ a=2, b=1$

Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

PRACTICE: Given the hyperbola $\frac{y^2}{100} - \frac{x^2}{139} = 1$, find the length of the a-axis and b-axis.

$(\boldsymbol{A})$ $a = 100, b = 139$
$(\boldsymbol{B})$ $a = 139, b = 100$
$(\boldsymbol{C})$ $a = \sqrt{139}, b = 10$
$(\boldsymbol{D})$ $a = 10, b = \sqrt{139}$

Circle	Ellipse	Parabola	**Hyperbola**

TOPIC: CONIC SECTIONS

Vertices and Foci of Hyperbolas

- Every hyperbola has 2 **Vertices** & 2 **Foci**, both located on the **[MAJOR | MINOR]** axis.
  - **Vertices** are the points on the hyperbola ___________ to the center

    Distance between center & **Vertex** = **[a | b | c]**
  - For any point on a hyperbola, the ___________ of the distances between the point & each **Focus** is a constant

    Distance between center & **Focus** = **[a | b | c]**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(*Horiz. Hyperbola*)

$$c^2 = a^2 - b^2$$

(*Foci of Ellipse*)

$$c^2 = a^2 \qquad b^2$$

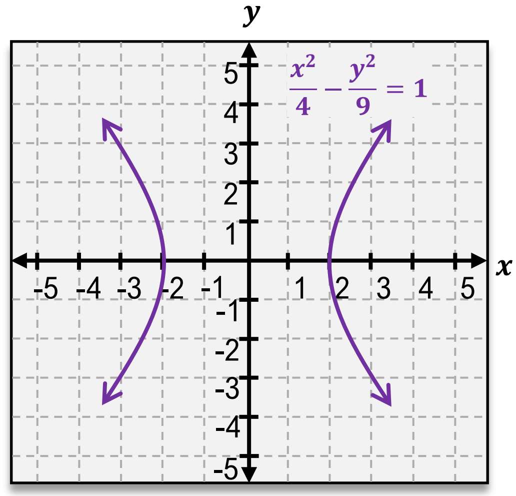
(*Foci of Hyperbola*)

EXAMPLE: Find the vertices & foci of the hyperbola in the graph.

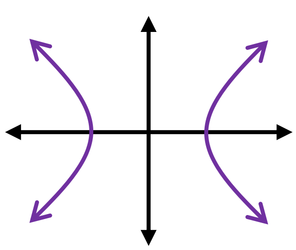
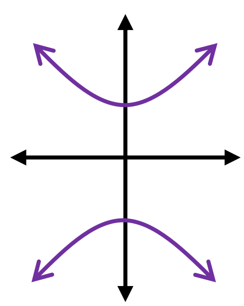
$a =$

$b =$

$c =$



- For a *vertical* hyperbola, the coordinates of vertices & foci are different

Horizontal Hyperbola	Vertical Hyperbola
	
Vertices: (___ ,0) & (___ ,0)	Vertices: (0 , ___) & (0 , ___)
Foci: (___ , 0) & (___ , 0)	Foci: (0, ___) & (0, ___)
Vertices & Foci on **[x | y]** axis	Vertices & Foci on **[x | y]** axis

Circle	Ellipse	Parabola	**Hyperbola**

<u>TOPIC: CONIC SECTIONS</u>

PRACTICE**:** Determine the vertices and foci of the hyperbola $\frac{y^2}{4} - x^2 = 1$.

$(\boldsymbol{A})$ Vertices: $(2,0), (-2,0)$
Foci: $(\sqrt{5}, 0), (-\sqrt{5}, 0)$

$(\boldsymbol{B})$ Vertices: $(0,2), (0,-2)$
Foci: $(0, \sqrt{5}), (0, -\sqrt{5})$

$(\boldsymbol{C})$ Vertices: $(1,0), (-1,0)$
Foci: $(5,0), (-5,0)$

$(\boldsymbol{D})$ Vertices: $(0,1), (0,-1)$
Foci: $(0,5), (0,-5)$

PRACTICE**:** Find the equation for a hyperbola with a center at $(0,0)$, focus at $(0,-6)$ and vertex at $(0,4)$.

$(\boldsymbol{A})$ $\frac{y^2}{16} - \frac{x^2}{20} = 1$

$(\boldsymbol{B})$ $\frac{y^2}{20} - \frac{x^2}{16} = 1$

$(\boldsymbol{C})$ $\frac{y^2}{4} - \frac{x^2}{\sqrt{20}} = 1$

$(\boldsymbol{D})$ $\frac{y^2}{\sqrt{20}} - \frac{x^2}{4} = 1$

Circle	Ellipse	Parabola	Hyperbola

TOPIC: CONIC SECTIONS

Asymptotes of Hyperbolas

- To graph hyperbolas, you'll need asymptotes
  - The values of a & b form a ______ where the asymptotes are draw through the corners of the shape.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

(*Vert. Hyperbola*)

$$y = \pm \frac{\quad}{\quad} x$$

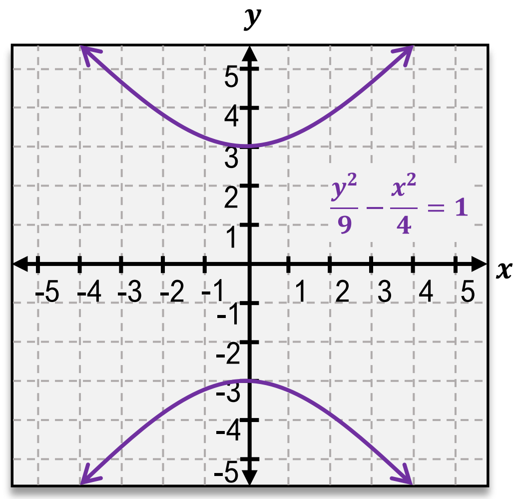
(Vert. *Hyperbola Asym.*)

EXAMPLE: Find & draw the asymptotes of the given hyperbola.

$a =$

$b =$

Asymptotes: $y =$ _________ & $y =$ _________



- For asymptotes of horizontal hyperbolas, just flip a & b

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(*Horiz. Hyperbola*)

$$y = \pm \frac{\quad}{\quad} x$$

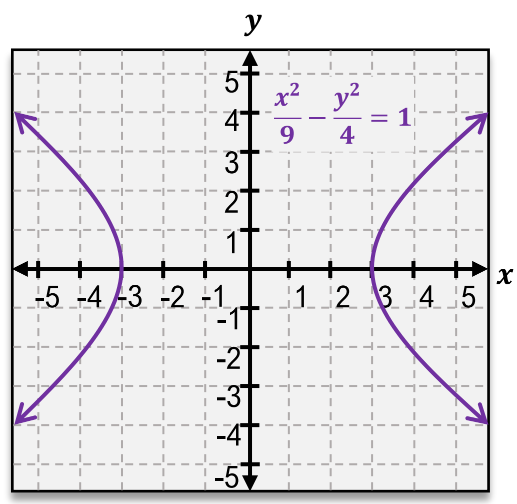
(Horiz. *Hyperbola Asym.*)

EXAMPLE: Find & draw the asymptotes of the given hyperbola.

$a =$

$b =$

Asymptotes: $y =$ _________ & $y =$ _________



PRACTICE: Find the equations for the asymptotes of the hyperbola $\frac{x^2}{64} - \frac{y^2}{100} = 1$.

$(\boldsymbol{A})\ y = \pm\frac{4}{5}x$

$(\boldsymbol{B})\ y = \pm\frac{5}{4}x$

$(\boldsymbol{C})\ y = \pm\frac{16}{25}x$

$(\boldsymbol{D})\ y = \pm\frac{25}{16}x$

Circle	Ellipse	Parabola	**Hyperbola**

TOPIC: CONIC SECTIONS

PRACTICE**:** Find the equations for the asymptotes of the hyperbola $\frac{y^2}{16} - \frac{x^2}{9} = 1$.

$(\boldsymbol{A})\ y = \pm\frac{9}{16}x$
$(\boldsymbol{B})\ y = \pm\frac{16}{9}x$
$(\boldsymbol{C})\ y = \pm\frac{3}{4}x$
$(\boldsymbol{D})\ y = \pm\frac{4}{3}x$

Circle	Ellipse	Parabola	**Hyperbola**

TOPIC: CONIC SECTIONS

EXAMPLE**:** Graph the hyperbola and identify the foci.

$$\frac{x^2}{9} - \frac{y^2}{64} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

TO GRAPH

1) Hyperbola is **[** HORIZONTAL **|** VERTICAL **]**

2) **Vertices** $(\pm a, 0)$, *OR* $(0, \pm a)$**:**

(__ , __) & (__ , __)

3) ***b* points** $(\pm b, 0)$, *OR* $(0, \pm b)$**:**

(__ , __) & (__ , __)

4) Asymptotes:

(A) draw a box through **vertices** & ***b* points**

(B) draw lines through box corners

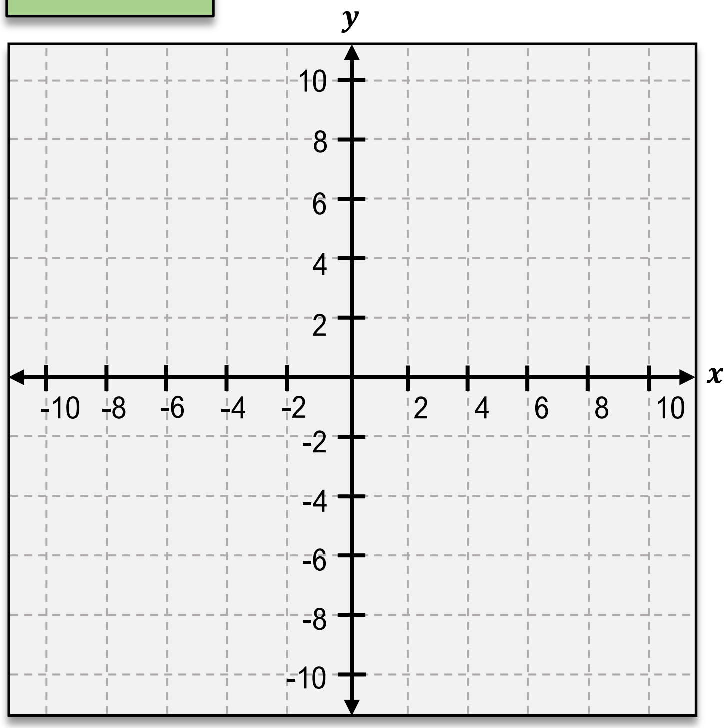
5) Draw branches at **vertices** & approaching asym.

FROM GRAPH

6) Foci $(\pm c, 0)$, *OR* $(0, \pm c)$**:**

$$c^2 = a^2 + b^2$$

(__ , __) & (__ , __)



Circle	Ellipse	Parabola	**Hyperbola**

TOPIC: CONIC SECTIONS

EXAMPLE**:** Graph the hyperbola and identify the foci.

$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$

TO GRAPH

1) Hyperbola is **[** HORIZONTAL **|** VERTICAL **]**

2) **Vertices** $(\pm a, 0)$, *OR* $(0, \pm a)$**:**

(__ , __) & (__ , __)

3) ***b* points** $(\pm b, 0)$, *OR* $(0, \pm b)$**:**

(__ , __) & (__ , __)

4) Asymptotes:

(A) draw a box through **vertices** & ***b* points**

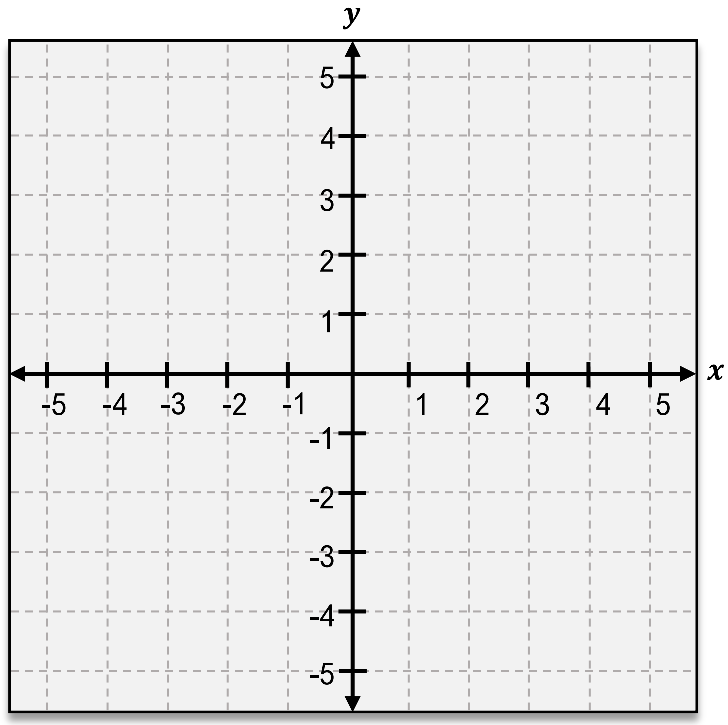
(B) draw lines through box corners

5) Draw branches at **vertices** & approaching asym.

FROM GRAPH

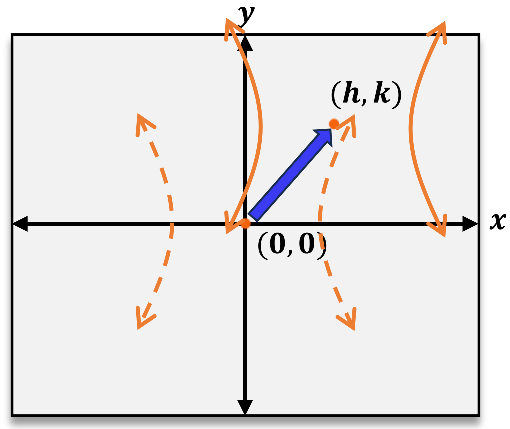
6) Foci $(\pm c, 0)$, *OR* $(0, \pm c)$**:**

(__ , __) & (__ , __)



TOPIC: CONIC SECTIONS
Graphing Hyperbolas NOT At The Origin

- To graph hyperbolas NOT at the origin, shift points by (h, k)



Horizontal Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \rightarrow \quad \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

EXAMPLE: Graph the following hyperbola.

$$\frac{(y-1)^2}{9} - \frac{(x-2)^2}{16} = 1$$

TO GRAPH

1) Hyperbola is **[HORIZONTAL | VERTICAL]**

2) Center (h, k): (___ , ___)

3) **Vertices** horiz. → $(h \pm a, k)$, *OR* vert. → $(h, k \pm a)$:

(__ , __) & (__ , __)

4) ***b*** **points** horiz. → $(h, k \pm b)$, *OR* vert. → $(h \pm b, k)$:

(__ , __) & (__ , __)

5) Asymptotes:

(A) draw a box through **vertices** & ***b*** **points**

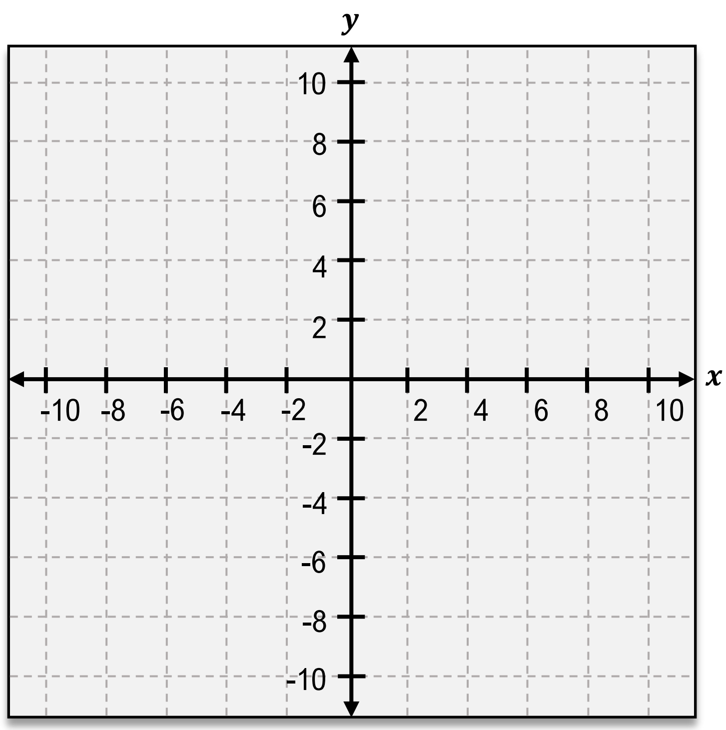
(B) draw lines through box corners

6) Draw branches at **vertices** & approaching asym.

FROM GRAPH

6) Foci horiz. → $(h \pm c, k)$, *OR* vert. → $(h, k \pm c)$:

(__ , __) & (__ , __)



Circle	Ellipse	Parabola	**Hyperbola**

TOPIC: CONIC SECTIONS

PRACTICE: Describe the hyperbola $\frac{(x+2)^2}{9} - \frac{(y-4)^2}{16} = 1$.

$(\boldsymbol{A})$ This is a *vertical* hyperbola centered at $(-2,4)$ with vertices at $(4,2), (4,-6)$ and foci at $(4,4), (4,-8)$.
$(\boldsymbol{B})$ This is a *vertical* hyperbola centered at $(2,-4)$ with vertices at $(4,1), (4,-5)$ and foci at $(4,3), (4,-7)$.
$(\boldsymbol{C})$ This is a *horizontal* hyperbola centered at $(-2,4)$ with vertices at $(2,4), (-6,4)$ and foci at $(4,4), (-8,4)$.
$(\boldsymbol{D})$ This is a *horizontal* hyperbola centered at $(-2,4)$ with vertices at $(1,4), (-5,4)$ and foci at $(3,4), (-7,4)$.

PRACTICE: Describe the hyperbola $y^2 - \frac{(x-1)^2}{4} = 1$.

$(\boldsymbol{A})$ This is a *vertical* hyperbola centered at $(1,0)$ with vertices at $(1,1), (1,-1)$ and foci at $(1,\sqrt{5}), (1,-\sqrt{5})$.
$(\boldsymbol{B})$ This is a *vertical* hyperbola centered at $(1,0)$ with vertices at $(1,2), (1,-2)$ and foci at $(1,1), (1,-1)$.
$(\boldsymbol{C})$ This is a *horizontal* hyperbola centered at $(-1,0)$ with vertices at $(0,0), (-2,0)$ and foci at $(\sqrt{5}-1,0), (-\sqrt{5}-1,0)$.
$(\boldsymbol{D})$ This is a *horizontal* hyperbola centered at $(1,0)$ with vertices at $(0,0), (-2,0)$ and foci at $(1,\sqrt{5}), (1,-\sqrt{5})$.