TOPIC: INTRODUCTION TO POWER SERIES

Intro to Power Series

◆ A power series is an infinite series that also involves a variable, ____, & can be thought of as an "infinite ____________."

► Power series have _____________ determined by c_n and __________ at $x = a$.

EXAMPLE For $c_n = \frac{1}{n!}$, write the first 4 terms of the $(\boldsymbol{A})$ infinite series & $(\boldsymbol{B})$ power series centered at $a = 2$.

Recall **Infinite Series**	New **Power Series**
$\sum_{n=0}^{\infty} c_n = c_0 + c_1 + c_2 + \cdots$	$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$
$\sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{\ \ !} + \frac{1}{\ \ !} + \frac{1}{\ \ !} + \frac{1}{\ \ !} + \cdots$ $= \quad + \quad + \quad + \quad + \cdots$	

◆ We can determine convergence of, perform operations on, and represent functions with power series.

EXAMPLE Determine where the power series are centered and list the four first terms of the given series.

$(\boldsymbol{A})$

$$\sum_{n=0}^{\infty} \frac{(x+1)^n}{(n+1)\cdot 3^n}$$

$(\boldsymbol{B})$

$$\sum_{n=0}^{\infty} (-1)^n n!\, x^n$$

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Radius of Convergence

◆ A power series only converges for certain values of x which is an interval determined by $|x - a| < R$.

► The radius of convergence ___ tells us how far away from the center ___ that the series converges.

EXAMPLE Find the radius of convergence for the following series.

$$\sum_{n=0}^{\infty} \frac{n \cdot (x-2)^n}{3^n}$$

HOW TO: Find Radius of Convergence

1) Apply **convergence** test

Ratio: $\lim_{n\to\infty} \left| \frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n} \right| < 1$

Root: $\lim_{n\to\infty} \sqrt[n]{|c_n(x-a)^n|} < 1$

Geometric: $\sum ar^n; |r| < 1$

2) Put in $|x - a| < R$ form to find R

R can be:

► a __________ value

► _______ if the limit = _______

► _______ if the limit = _______

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EXAMPLE Find the radius of convergence of the series.

$(\boldsymbol{A})$

$$\sum_{n=0}^{\infty} \frac{2^n (x-1)^n}{n!}$$

$(\boldsymbol{B})$

$$\sum_{n=0}^{\infty} n^n x^n$$

$(\boldsymbol{C})$

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n4^n}$$

HOW TO: Find Radius of Convergence

1) Apply **convergence** test

Ratio: $\lim\limits_{n\to\infty} \left|\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}\right| < 1$

Root: $\lim\limits_{n\to\infty} \sqrt[n]{|c_n(x-a)^n|} < 1$

Geometric: $\sum ar^n; |r| < 1$

2) Put in $|x-a| < R$ form to find R

R can be:

- a **finite** value
- $\mathbf{0}$ if the limit = ∞
- ∞ if the limit = $\mathbf{0}$

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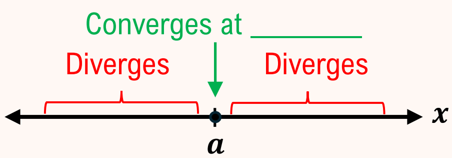
Interval of Convergence

- Recall: Radius of convergence $\boldsymbol{R}$ is how far from center $\boldsymbol{a}$ that a power series converges.
  - The *interval* of convergence is found by ____________ the inequality $|x - a| < R$.

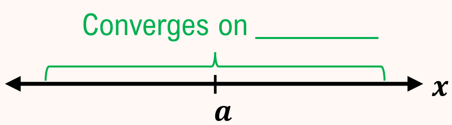
EXAMPLE Find the interval of convergence of the power series below given its radius of convergence.

New | **Interval of Convergence**

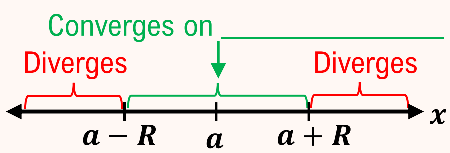
$\boldsymbol{R = 0}$



$\boldsymbol{R = \infty}$



$\boldsymbol{R = finite\ \#}$



$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n4^n}$$

Radius of Convergence: $R = 4$

Inequality: $|x + 3| < 4$

- We must also determine convergence of ____________ by plugging them in for x into the original power series.

EXAMPLE Determine endpoint convergence for the power series.

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n4^n}$$

$x = $ ___: $\displaystyle\sum_{n=1}^{\infty} \frac{(\quad +3)^n}{n4^n}$

$x = $ ___: $\displaystyle\sum_{n=1}^{\infty} \frac{(\quad +3)^n}{n4^n}$

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EXAMPLE Find the interval of convergence of each series.

$(\boldsymbol{A})$

$$\sum_{k=0}^{\infty} \frac{k!\, x^{2k}}{3^k}$$

Recall

Ratio: $\lim_{n\to\infty} \left|\frac{c_{n+1}(x-a)^{n+1}}{c_n(x-a)^n}\right| < 1$

Root: $\lim_{n\to\infty} \sqrt[n]{|c_n(x-a)^n|} < 1$

Geometric: $\sum ar^n; |r| < 1$

$(\boldsymbol{B})$

$$\sum_{n=1}^{\infty} \frac{n!}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)} x^n$$

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Representing Functions as Power Series

◆ Representing complicated functions as power series lets us *approximate & manipulate* them more easily.

► To do this, we often use the form of a geometric series:

Recall

$$S = \sum_{n=0}^{\infty} a\, r^n = \frac{a}{1-r}$$

EXAMPLE

Find a power series for the function $f(x) = \frac{1}{x}$ centered at $a = 1$ and determine the interval of convergence.

HOW TO: Represent $f(x)$ as a Power Series

1) Write f in the form:

2) Identify a & r

3) Plug a & r into $\sum_{n=0}^{\infty} a\, r^n$

4) Find int. of convergence w/ *geometric series test:* $|\quad| < 1$

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EXAMPLE

Find the power series representation centered at $x = 0$ of the following function. Give the interval of convergence for the resulting series.

$$f(x) = \frac{2}{2+x}$$

HOW TO: Represent $f(x)$ as a Power Series

1) Write f in the form: $\frac{a}{1-r}$

2) Identify a & r

3) Plug a & r into $\sum_{n=0}^{\infty} a\, r^n$

4) Find int. of convergence w/ *geometric series test:* $|r| < 1$

PRACTICE

Find the power series representation centered at $x = 0$ of the following function. Give the interval of convergence for the resulting series.

$$f(x) = \frac{1}{1-x^3}$$