TOPIC: CALCULUS WITH PARAMETRIC CURVES

Differentiation of Parametric Curves

◆ Recall: Parametric equations express x and y as functions of the parameter t.

► The *derivative* of a parametric equation is found by ____________ the *derivatives* of y & x with respect to ___.

EXAMPLE Find the derivative of the curve defined by the equations $\boldsymbol{x(t) = t^2 + 3t}$ & $\boldsymbol{y(t) = 2t^3 - 4}$.

New **Derivative of a Parametric Curve**

$$\frac{dy}{dx} = \frac{\quad}{\quad} = \frac{\quad}{\quad}; \quad x'(t) \neq 0$$

$y'(t) =$

$x'(t) =$

$\frac{dy}{dx} =$

EXAMPLE Find the equation of the tangent line to the curve defined by the equations $\boldsymbol{x(t) = t^2 + 3t}$ & $\boldsymbol{y(t) = 2t^3 - 4}$ when $t = 1$.

Recall
$y - y_1 = m(x - x_1)$
(Point-Slope)

$x_1 = x(\quad) =$

$y_1 = y(\quad) =$

$m = \frac{dy}{dx}\Big|_{t=} \quad =$

Tangent line:

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EXAMPLE Find $\frac{dy}{dx}$ for the parametric curve given by $x(t) = 3\cos(t)$ and $y(t) = 3\sin(t)$.

PRACTICE Write the equation of the tangent line in cartesian coordinates for the given parameter t.

$$x = 8\cos t, \quad y = 6\sin t, \quad t = \frac{\pi}{4}$$

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EXAMPLE Find all the points on the curve $x(t) = t - \frac{1}{t}$, $y(t) = t + \frac{1}{t}$ that have a slope of 0.

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Higher-Order Derivatives of Parametric Curves

◆ To find the 2nd deriv. of a parametric equation, differentiate the 1st deriv. w/ respect to t and divide by ______.

EXAMPLE

Find ($\boldsymbol{A}$) the first derivative and ($\boldsymbol{B}$) the second derivative of the equations defined by $\boldsymbol{x(t) = t^2 + 3t}$ and $\boldsymbol{y(t) = 2t^3 - 4}$.

Recall: 1st Deriv. of Parametric Eqns	New: 2nd Derivative of Parametric Equations
$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)}$	$\frac{d^2y}{dx^2} = \frac{d/dt\left(\text{———}\right)}{\quad}$
$y'(t) = 6t^2$	$d/dt\left(\text{————}\right) =$ Recall: $\frac{d}{dt}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}$ *(Quotient Rule)*
$x'(t) = 2t + 3$	
$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{6t^2}{2t + 3}$	$\boldsymbol{\frac{dx}{dt}} =$
	$\frac{d^2y}{dx^2} =$

◆ To find any higher-order derivative, differentiate the ____________ derivative with respect to t and divide by ____.

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EXAMPLE

Determine the concavity of the curve $x = t, y = t^3 - 3t$. *Recall: A function is concave up if its 2^{nd} derivative is positive and concave down if its 2^{nd} derivative is negative.*

PRACTICE

Find $\frac{d^3y}{dx^3}$ for the parametric curve at the given point.

$x = 2t^3, \;\; y = t^4, \;\; t = 1$

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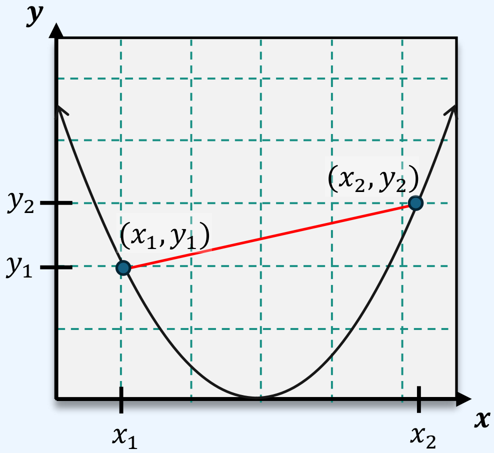
Arc Length of Parametric Curves

◆ The distance traveled along a curve between two points is called the *arc length*.

► Arc length s is the sum of distances of ____________ many line segments between two points on a curve.

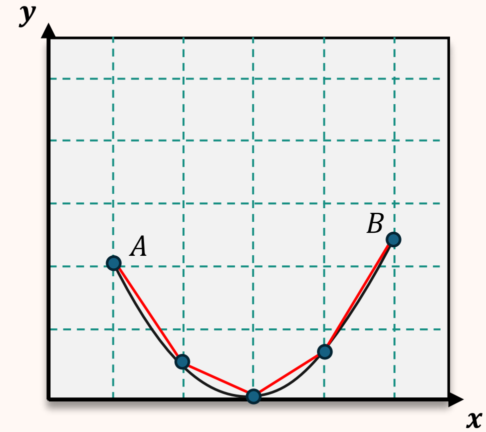
Recall — **Distance Between Two Points**



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(Distance Formula)

New — **Arc Length of Parametric Curves**



For a parametric curve defined by:

$x = x(t)$ & $y = y(t), a \le t \le b$:

$$s = \int_a^b \sqrt{\left(\frac{\quad}{\quad}\right)^{\;} + \left(\frac{\quad}{\quad}\right)^{\;}}\, dt$$

EXAMPLE Find the arc length of the curve defined by the equations $x(t) = 2\cos t$ and $y(t) = 2\sin t$ on the interval $[0, \pi]$.

$\frac{dx}{dt} =$

$\frac{dy}{dt} =$

$\int \sqrt{(\qquad) + (\qquad)}\, dt =$

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EXAMPLE Find the arc length of the parametric curve defined by the equations below, from $t = 0$ to $t = 1$.

$$x(t) = t^3, y(t) = 2t^2$$

PRACTICE Find the length of the curve below on the interval $[0,4]$.

$$x = \frac{1}{3}(2t + 3)^{\frac{3}{2}}, y = \frac{1}{2}t^2 + t$$