TOPIC: CALCULUS IN POLAR COORDINATES

Differentiation of Polar Curves

- Polar curves $r =$ ______ can be written in parametric form $x =$ ______________ & $y =$ ______________.
  - Polar cuves can be *differentiated* by converting to parametric form and using the chain rule.

EXAMPLE Find the slope of the tangent line of the polar curve $r = 2\sin\theta$ at $\theta = \pi/6$.

New **Slope of Polar Curves**

$$\frac{dy}{dx} = \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ ; \qquad \frac{dx}{d\theta} \neq 0$$

Convert to parametric form:

$y =$

$x =$

Differentiate the parametric equations:

$\frac{dy}{d\theta} =$

$\frac{dx}{d\theta} =$

Find $\frac{dy}{dx}$:

$\frac{dy}{dx} =$

Plug in $\theta =$:

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EXAMPLE Find the derivative $\frac{dy}{dx}$ for the polar curve $r = 2 \cdot \sin(\theta)$.

PRACTICE Find the slope of the tangent line to the polar curve $r = 2\cos(\theta)$ at $\theta = \frac{\pi}{3}$.

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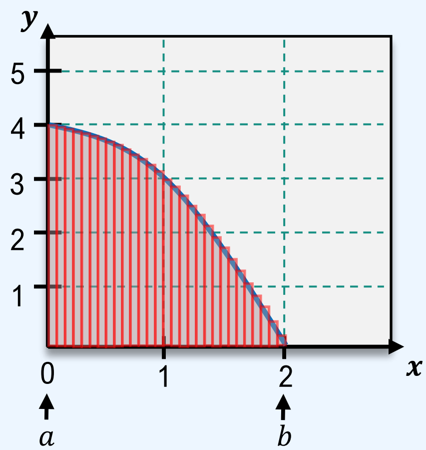
Area of Polar Regions

◆ Recall: The area under a curve $f(x)$ is the sum of the area of infinite *rectangles*, found using a definite integral.

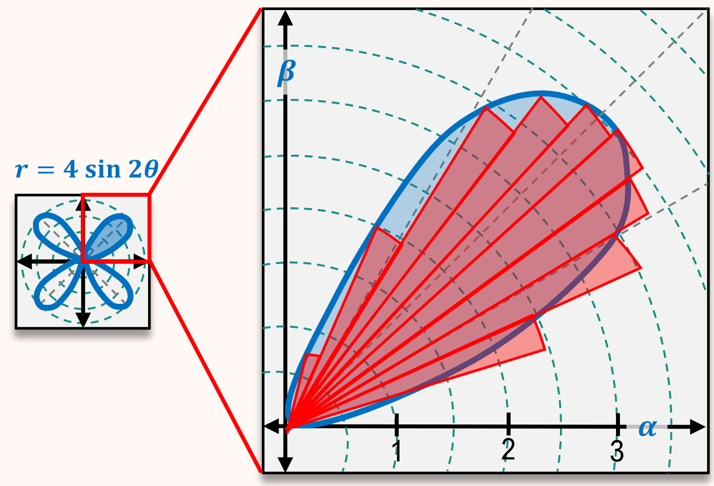
► The area of a polar region $r = f(\theta)$ is the sum of infinite *sector areas* $A = \frac{1}{2}r^2\theta$, also found using a def int.

Recall — **Area Under a Curve**



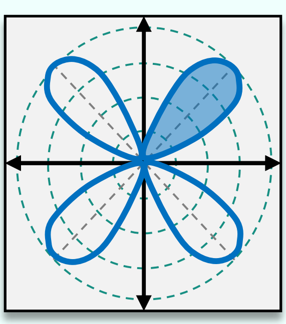
$$\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k^*)\Delta x = \int_a^b f(x)\,dx$$

New — **Area of Polar Regions**



$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{2}f(\theta_k)^2\Delta\theta = \text{—}\cdot\int_\alpha^\beta [\qquad]\,d\theta = \text{—}\cdot\int_\alpha^\beta [\quad]\,d\theta$$

EXAMPLE Find the area of one petal of the rose defined by $r = 4\sin 2\theta$.



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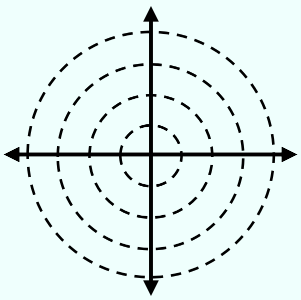
EXAMPLE Find the area enclosed by the inner loop of the limaçon given by $r = 1 - 2\cos\theta$.



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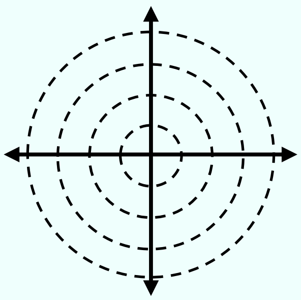
EXAMPLE Find the area enclosed by one loop of the lemniscate given by $r^2 = 4\sin 2\theta$.



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PRACTICE Find the area enclosed by the cardioid $r = 2 - 2\sin\theta$ between $\theta = \frac{\pi}{6}$ and $\theta = \frac{2\pi}{3}$.