TOPIC: PARAMETRIC EQUATIONS

Introduction to Parametric Equations

◆ **Parametric Equations** express *two* variables (x & y) in terms of a _____ (usually t), called the **parameter**.

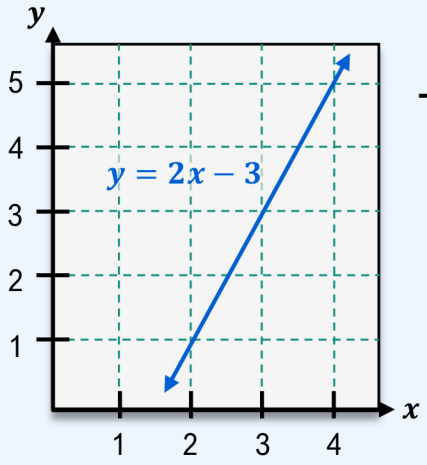
- To graph, make a table of values with ___, x, & y.

EXAMPLE Graph the equations.

Recall **Two-Variable Equations**

[1 | 2 | 3] equation(s)

[1 | 2 | 3] variable(s)

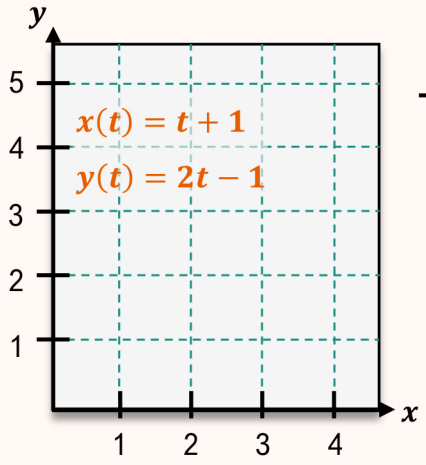


x	y
2	1
3	3
4	5

New **Parametric Equations**

[1 | 2 | 3] equation(s)

[1 | 2 | 3] variable(s)



t	$x(t)$	$y(t)$
1		
2		
3		

◆ Graphs of parametric eq'ns are called **plane curves**. Indicate orientation with arrows along direction of ___ t.

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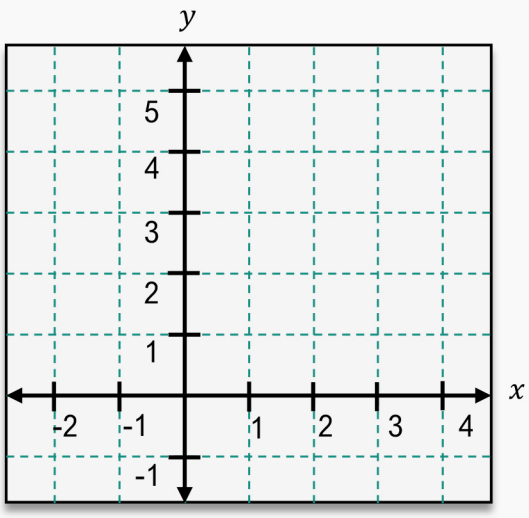
PRACTICE Graph the plane curve formed by the parametric equations and indicate its orientation.

$$x(t) = -t + 1; \qquad y(t) = t^2$$

$$-2 \le t \le 2$$

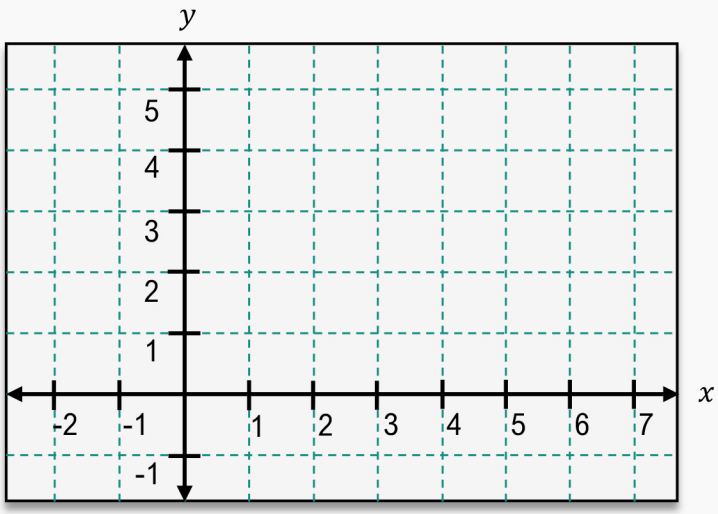


t	x	y

PRACTICE Graph the plane curve formed by the parametric equations and indicate its orientation.

$$x(t) = 2t - 1; \qquad y(t) = 2\sqrt{t}$$

$$t \ge 0$$



t	x	y

TOPIC: PARAMETRIC EQUATIONS

Eliminating the Parameter

◆ You'll often have to graph parametric eq'ns and "**eliminate the (t) parameter**", leaving an eq'n with *only* x & y.

- Given $x(t)$ & $y(t)$, solve _____ eq'n for t, then substitute.

New **Eliminate the Parameter**

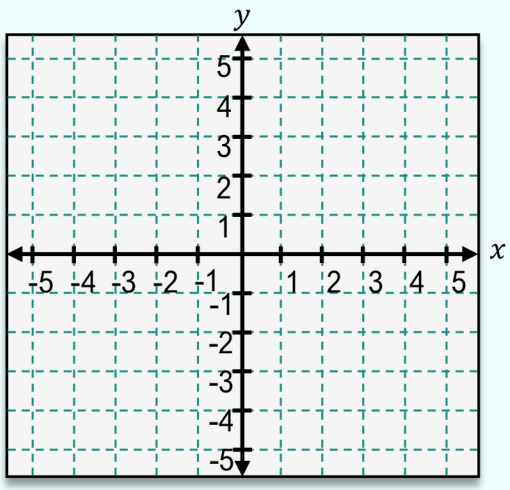
$x = t - 2$

$y = 3t^2$

$y = 3(\_\_\_\_\_)^2$

EXAMPLE Graph the parametric equations, then eliminate the parameter to convert them to a rectangular equation.

$x(t) = \sqrt{t}, \quad y(t) = t - 3; \quad t \geq 0$



t	$x(t)$	$y(t)$
0		
1		
2		
3		
4		

◆ **Note:** If given parametric eq'n restrict t interval, the graph is usually a small portion of the rectangular eq'n.

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PRACTICE Eliminate the parameter to rewrite the following as a rectangular equation.

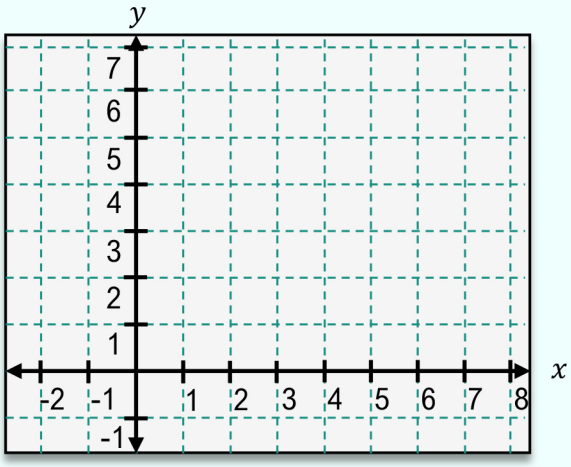
$x(t) = 2t - 1$

$y(t) = t^5 - 2$

EXAMPLE Graph the plane curve of the parametric equations. Write the equivalent rectangular equation.

$$x(t) = t^2 - 1; \qquad y(t) = t^2 - 2$$

$$-\infty \leq t \leq \infty$$



t	x	y

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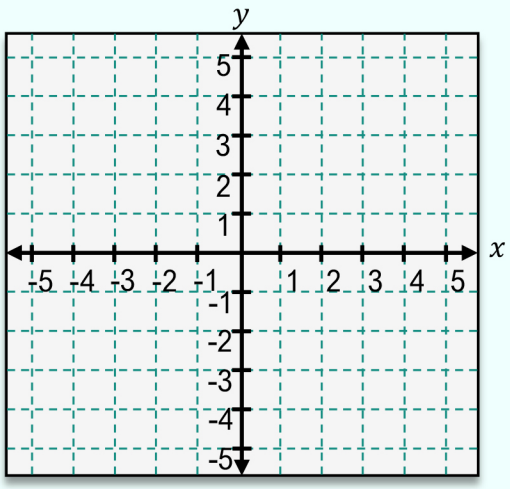
EXAMPLE Graph the plane curve of the parametric equations. Write the equivalent rectangular equation.

$$x(t) = t - 3; \qquad y(t) = \frac{1}{t-5}$$

$$t \neq 5$$



t	x	y
0		
3		
4.8		
5.2		
7		

TOPIC: PARAMETRIC EQUATIONS

Equations with Trigonometriic Functions

◆ To eliminate the parameter from $x(t)$ & $y(t)$ with trig, rewrite ***BOTH*** eq'ns and use a Pythagorean Identity.

EXAMPLE Eliminate the parameter.

Recall **Eliminate the Parameter**

*Solve **ONE** eq'n for t*

$$x = t - 2 \quad \Rightarrow \quad y = 3(x+2)^2$$
$$y = 3t^2$$

New **Eliminate Parameter with Trig**

*Solve **BOTH** eqn's for* _____

$$x = \cos t$$
$$y = 3\sin t$$

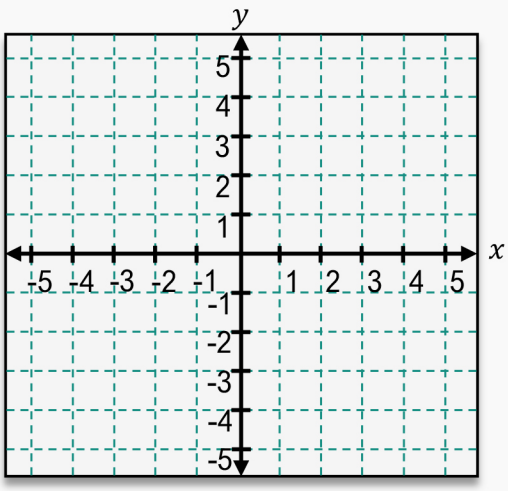
Recall

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

(Pythagorean Identities)

PRACTICE First eliminate the parameter, then graph the plane curve of the parametric equations.

$$x(t) = 2 + \cos t \quad y(t) = -1 + \sin t; \qquad 0 \le t \le 2\pi$$



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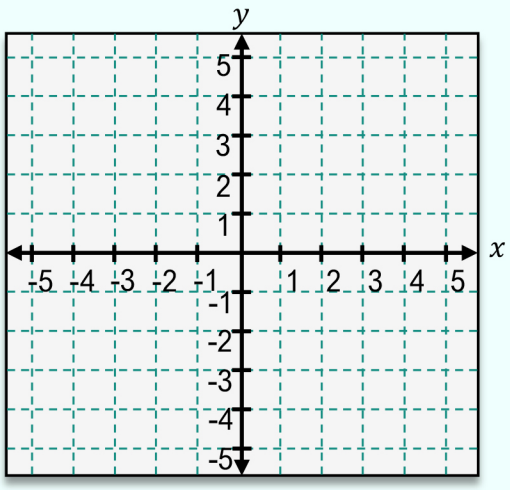
EXAMPLE First eliminate the parameter, then graph the plane curve of the parametric equations.

$$x(t) = 3\cos t; \qquad y(t) = 2\sin t$$

$$\pi \leq t \leq 2\pi$$



TOPIC: PARAMETRIC EQUATIONS

Parameterize Equations

◆ Often you'll be given an eq'n with only x & y and be asked to write $x(t)$ & $y(t)$ → **"parameterize"**.

- ***Choose*** a t (avoid domain restrictions), solve for $x(t)$, then _____ into original eq'n to find $y(t)$.

Recall — Eliminate the Parameter

$$\left.\begin{aligned} x(t) &= t - 1 \\ y(t) &= 4t \end{aligned}\right\} \; y = 4(x+1)$$

New — Parameterize Equations

$$y = 4(x+1) \;\Leftrightarrow\; \begin{cases} t = \\ x(t) = \\ y(t) = \end{cases} \quad \text{OR} \quad \begin{cases} t = \\ x(t) = \\ y(t) = \end{cases}$$

TRY

$t = x$

$t = \# \cdot x,$

$t = x \pm \#$

AVOID

$t = \# \cdot x^2$

$t = \sqrt{x}$

EXAMPLE Without choosing $x(t) = t$, find parametric equations for the rectangular equation.

$(\boldsymbol{A})$

$$y = 2x + 5$$

$t = x + 1$

$(\boldsymbol{B})$

$$y = (x+2)^2 - 3$$

HOW TO: Parameterize Equations

1) Define t, unless given

Try $t = x$, or $t = (\_\_\_)$ if () in eq'n

2) Solve for $x(t)$

3) Plug $x(t)$ into original eq'n for x

4) Solve for $y(t)$

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EXAMPLE Write two sets of parametric equations for the rectangular eq'n below.

$$y = 2x^3$$

HOW TO: Parameterize Equations

1) Define t, unless given

Try $t = x$, or $t =$ (___) if () in eq'n

2) Solve for $x(t)$

3) Plug $x(t)$ into original eq'n for x

4) Solve for $y(t)$

TOPIC: PARAMETRIC EQUATIONS
Equations of Circles and Ellipses

◆ To parameterize an eq'n containing $x^2 + y^2$, write the eq'n in the form (_____)^2 + (_____)^2 = 1.

- Set $f(x)^2 =$ _____(t), and $g(y)^2 =$ _____(t), then solve for x & y.

Recall — Eliminate the Parameter

$x(t) = 2\cos t$

$y(t) = \sin t$

$\cos^2(t) + \sin^2(t) = 1$

$\left(\frac{x}{2}\right)^2 + y^2 = 1$

New — Parameterize Equations

$\left(\frac{x}{2}\right)^2 + y^2 = 1$

$x(t) =$

$y(t) =$

EXAMPLE Write parametric equations for the equation below.

$9x^2 + y^2 = 9$

HOW TO: Parameterize Equations

1) Write eq'n as $f(x)^2 + g(y)^2 = 1$

2) Set $f(x)^2 = \cos^2(t)$, solve for $x(t)$

3) Set $g(y)^2 = \sin^2(t)$, solve for $y(t)$

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PRACTICE Write parametric equations for the rectangular equation below.

$$x^2 + y^2 = 25$$

HOW TO: Parameterize Equations

1) Write eq'n as $f(x)^2 + g(y)^2 = 1$

2) Set $f(x)^2 = \cos^2(t)$, solve for $x(t)$

3) Set $g(y)^2 = \sin^2(t)$, solve for $y(t)$