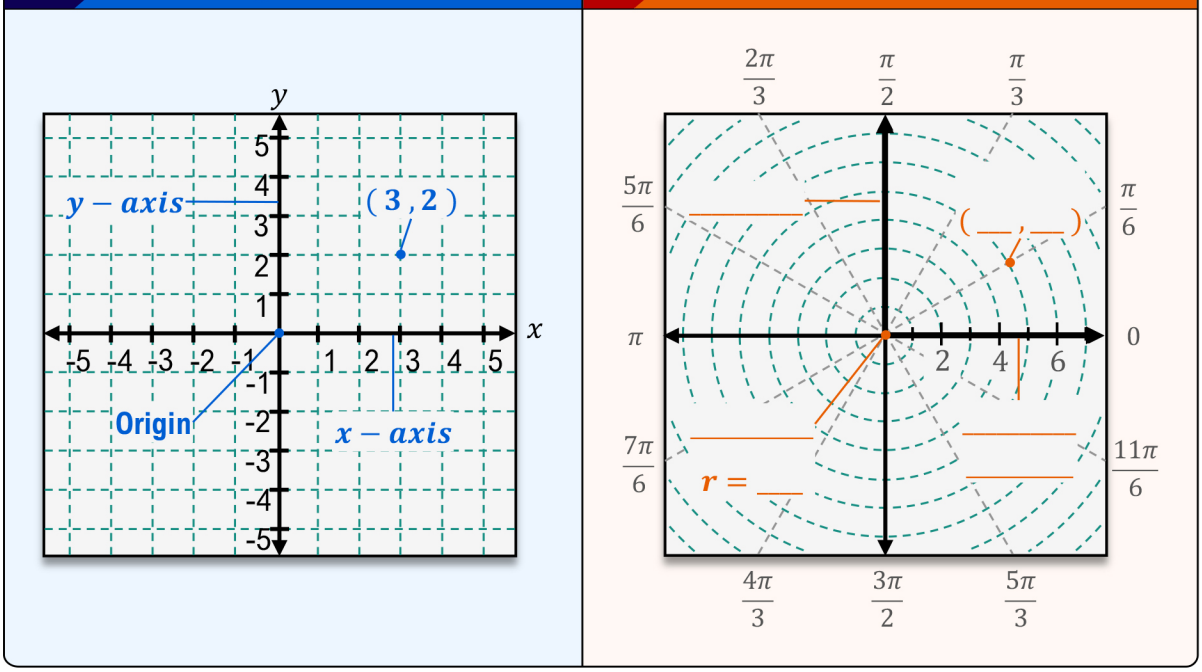
## TOPIC: POLAR COORDINATES

### Intro to Polar Coordinates

- Like the rectangular system, you'll use the **polar coordinate system** to plot points in terms of $\boldsymbol{r}$ & $\boldsymbol{\theta}$.
  - $\boldsymbol{r}$ is the distance from the **pole** (*origin*), and $\boldsymbol{\theta}$ is the $\angle$ from the **polar axis** ($+x$–*axis*).

**Recall** Rectangular Coordinates | **New** Polar Coordinates




- To plot points in polar coordinates, locate ____ *first*, then count ____ units away from pole.

**EXAMPLE** Plot the points on the polar coordinate system.

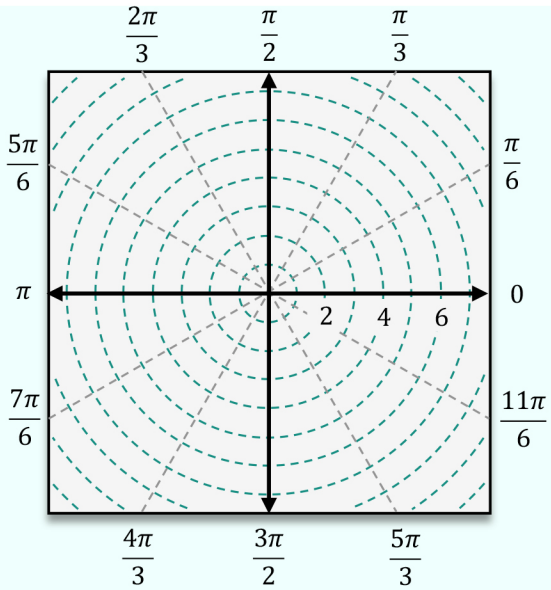
$(\boldsymbol{A})$ $\left(4, \frac{\pi}{3}\right)$

_____

$(\boldsymbol{B})$ $\left(5, -\frac{\pi}{3}\right)$

_____

$(\boldsymbol{C})$ $\left(-3, \frac{\pi}{6}\right)$

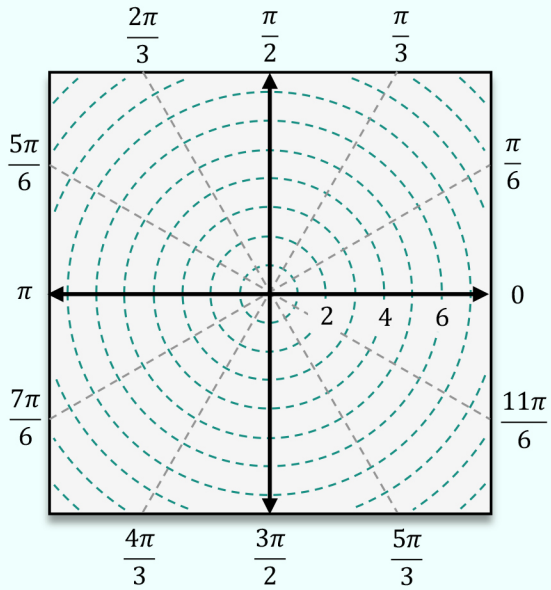



- If $\theta < 0$, measure angle **[CW | CCW]**. This is a reflection over the **[POLE | POLAR AXIS]**.
- If $r < 0$, count from pole in _____ direction. This is a reflection over the **[POLE | POLAR AXIS]**.

## TOPIC: POLAR COORDINATES

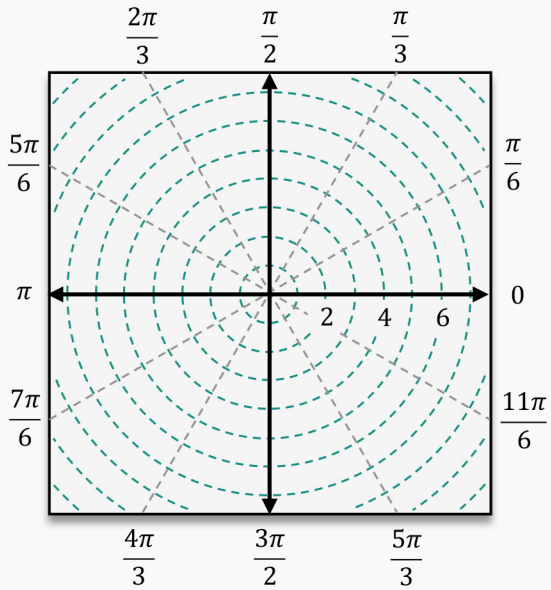
**EXAMPLE** Plot the points on the polar coordiante system.

| | |
|---|---|
| $(\boldsymbol{A})$ $(5, 60°)$ | $(\boldsymbol{B})$ $(3, 90°)$ |
| $(\boldsymbol{C})$ $\left(0, -\frac{5\pi}{3}\right)$ | $(\boldsymbol{D})$ $\left(2, \frac{7\pi}{3}\right)$ |




**PRACTICE** Plot the points on the polar coordinate system.

| | |
|---|---|
| $(\boldsymbol{A})$ $(5, 210°)$ | $(\boldsymbol{B})$ $(-3, -90°)$ |
| $(\boldsymbol{C})$ $\left(6, -\frac{11\pi}{6}\right)$ | $(\boldsymbol{D})$ $\left(-2, \frac{2\pi}{3}\right)$ |

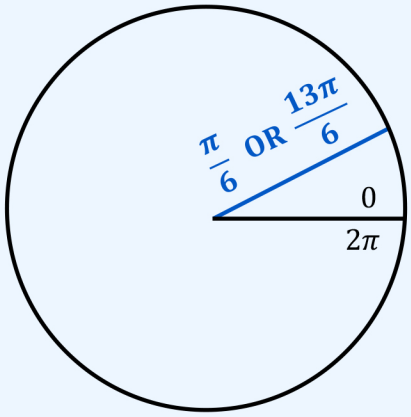


## TOPIC: POLAR COORDINATES

### Determining Different Coordinates for the Same Point

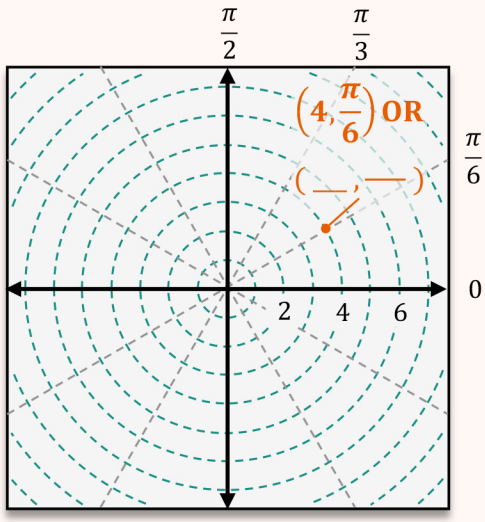
◆ In polar coordinates, *one* point can be represented by *multiple* ordered pairs.

**Recall** — **Coterminal Angles**




$$\theta_2 = \theta_1 \pm 2\pi n$$

**New** — **Diff. Coordinates for Same Point**




$(r, \theta) \rightarrow$ $(r, \theta \pm$ _____$)$

( __ $r, \theta +$ ___ $\pm 2\pi n)$

**EXAMPLE** For the point $(4, \frac{\pi}{6})$, find another set of coordinates, $(r, \theta)$, where:

$(\boldsymbol{A})$ $r \geq 0, \quad -2\pi \leq \theta \leq 0$

$(\boldsymbol{B})$ $r \leq 0, \quad 0 \leq \theta \leq 2\pi$

## TOPIC: POLAR COORDINATES

**PRACTICE** Plot the point $\left(3, \frac{\pi}{2}\right)$ & find another set of coordinates, $(r, \theta)$, for this point, where:

$(\boldsymbol{A})$

$r \geq 0, \qquad 2\pi \leq \theta \leq 4\pi$

---

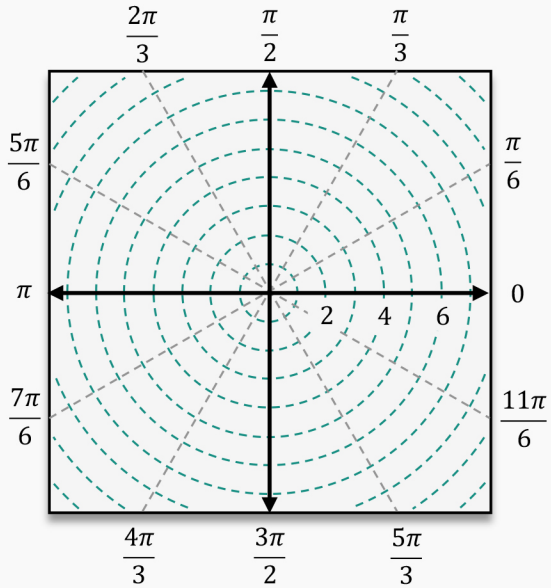
$(\boldsymbol{B})$

$r \geq 0, \qquad -2\pi \leq \theta \leq 0$

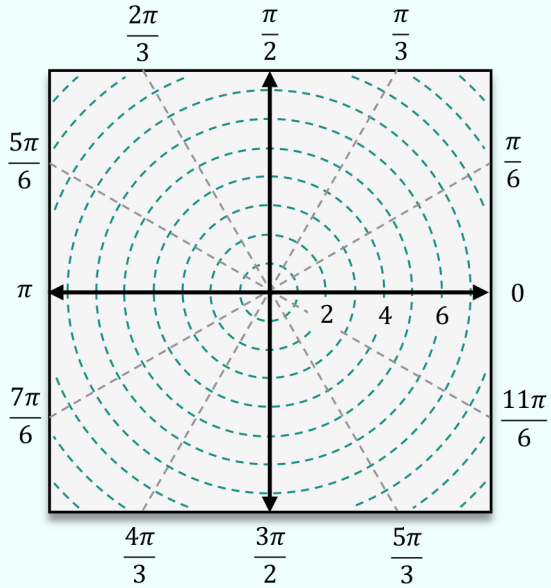
---

$(\boldsymbol{C})$

$r \leq 0, \qquad 0 \leq \theta \leq 2\pi$



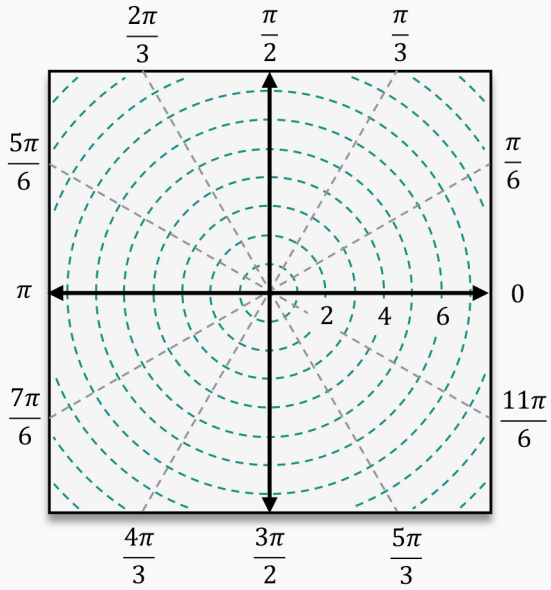

**EXAMPLE** Plot the point $\left(-2, \frac{5\pi}{3}\right)$ & find 3 sets of coordinates, $(r, \theta)$, for this point.



## TOPIC: POLAR COORDINATES

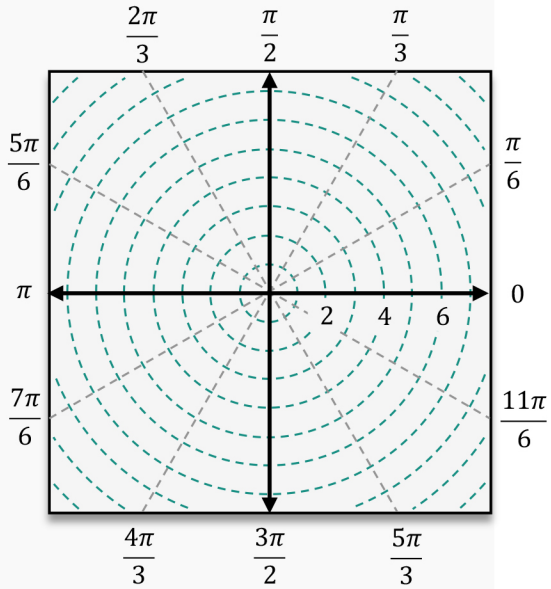
**PRACTICE** Plot the point $\left(5, -\frac{\pi}{3}\right)$, then identify which of the following sets of coordinates is the same point.

$\left(-5, -\frac{\pi}{3}\right)$ $\left(-5, \frac{\pi}{3}\right)$ $\left(-5, \frac{2\pi}{3}\right)$ $\left(-5, \frac{5\pi}{3}\right)$



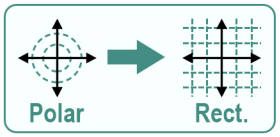

**PRACTICE** Plot the point $\left(-3, -\frac{\pi}{6}\right)$, then identify which of the following sets of coordinates is the same point.

$\left(-3, \frac{11\pi}{6}\right)$ $\left(-3, \frac{5\pi}{6}\right)$ $\left(3, \frac{11\pi}{6}\right)$ $\left(3, \frac{\pi}{6}\right)$



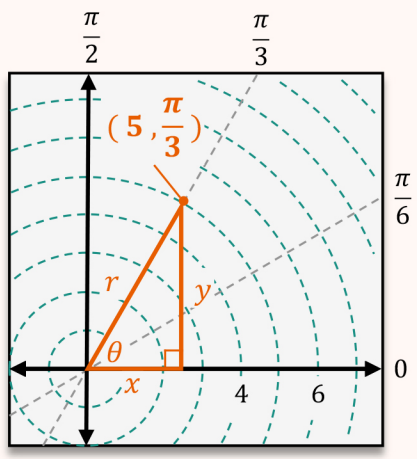
## TOPIC: POLAR COORDINATES

### Convert Points from Polar to Rectangular




◆ To convert polar coordinates to rectangular, use $\sin$ & $\cos$.

**New** **Polar → Rectangular Coordinates**

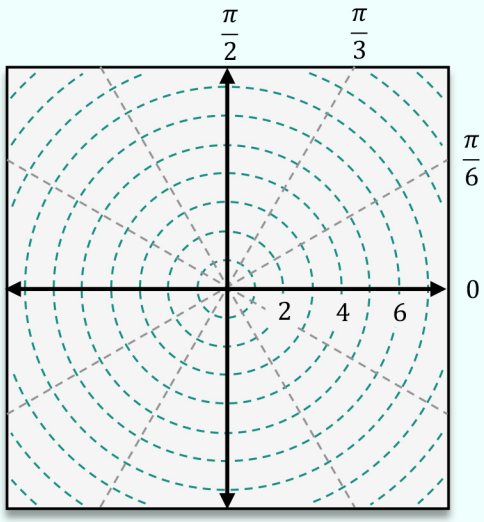



$\cos\theta = \dfrac{adj}{hyp}$ $\qquad$ $\sin\theta = \dfrac{opp}{hyp}$

$\cos\dfrac{\pi}{3} =$ ___ $\qquad$ $\sin\dfrac{\pi}{3} =$ ___

$x = r$_____ $\theta$ $\qquad$ $y = r$_____ $\theta$

**EXAMPLE** Plot the points on the polar coordinate system, then convert them into rectangular coordinates and plot in the $x - y$ plane.



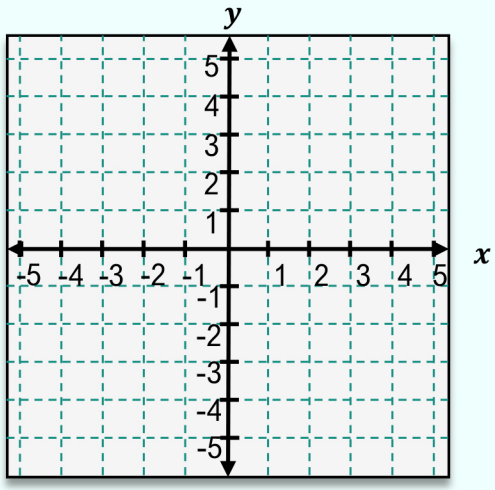

$(\boldsymbol{A})$

$\left(-3, \dfrac{\pi}{6}\right) \rightarrow$ (____, ____)

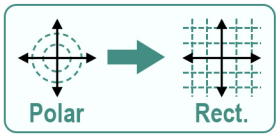
---

$(\boldsymbol{B})$

$\left(0, -\dfrac{\pi}{6}\right) \rightarrow$ (____, ____)



## TOPIC: POLAR COORDINATES




**PRACTICE** Convert the points to rectangular coordinates.

**Recall**

$x = r\cos\theta$

$y = r\sin\theta$

$(\boldsymbol{A})$

$\left(-2, -\frac{\pi}{4}\right)$

---

$(\boldsymbol{B})$

$\left(4, \frac{\pi}{6}\right)$

---

$(\boldsymbol{C})$

$(-3,0)$

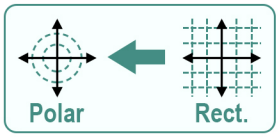
---

$(\boldsymbol{D})$

$\left(0, \frac{7\pi}{4}\right)$

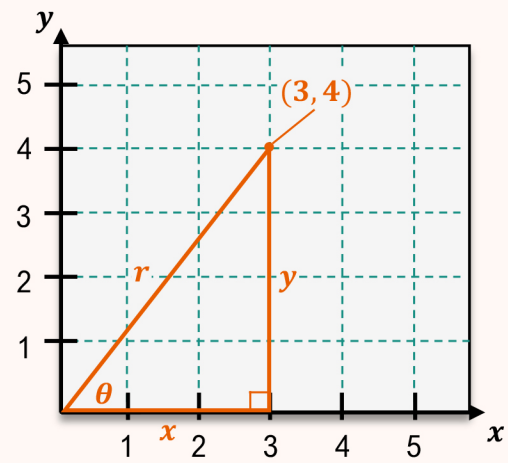
## TOPIC: POLAR COORDINATES

### Convert Points from Rectangular to Polar




◆ To convert rectangular coordinates to polar, use the Pythagorean Theorem and tan.

**New** — **Rectangular → Polar Coordinates**

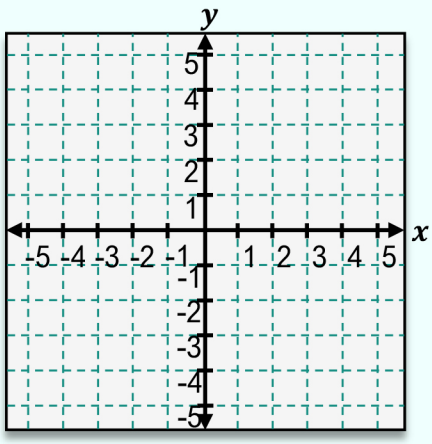



$c^2 = a^2 + b^2$ $\qquad \tan\theta = \dfrac{opp}{adj}$

$r^2 =$ ____ + ____ $\qquad \tan\theta =$ ____

**EXAMPLE** Plot the points, then convert them into polar coordinates.

$(A)$ $(-4,0) \rightarrow$ (____, ____)




$(B)$ $(-1, \sqrt{3}) \rightarrow$ (____, ____)

**HOW TO: Convert Points Rect. to Polar**

**1)** Plot the point $(x, y)$

**2)** Calculate $r = \sqrt{\_\_\_\_ + \_\_\_\_}$

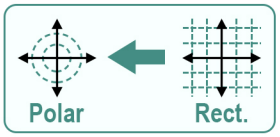
**3)** Calculate $\theta$. If point is…

On axis: $\theta = [\,0 \mid \frac{\pi}{2} \mid \pi \mid \frac{3\pi}{2}\,]$

In Q1 or Q4: $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

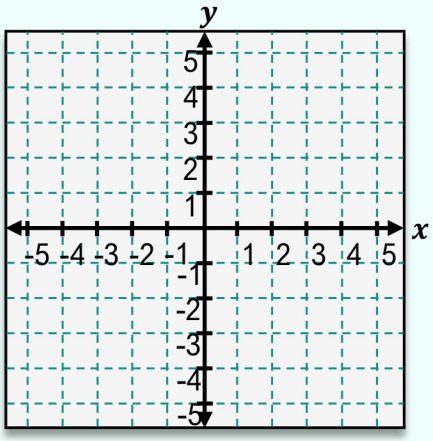
In Q2 or Q3: $\theta = \tan^{-1}\left(\frac{y}{x}\right) +$ ____

## TOPIC: POLAR COORDINATES




**EXAMPLE** Convert the point to polar coordinates.

$(3, -3)$




**HOW TO: Convert Points Rect. to Polar**

**1)** Plot the point $(x, y)$

**2)** Calculate $r = \sqrt{x^2 + y^2}$

**3)** Calculate $\theta$. If point is…

On axis: $\theta = [\,0 \mid \frac{\pi}{2} \mid \pi \mid \frac{3\pi}{2}\,]$

In Q1 or Q4: $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

In Q2 or Q3: $\theta = \tan^{-1}\left(\frac{y}{x}\right) + \pi$

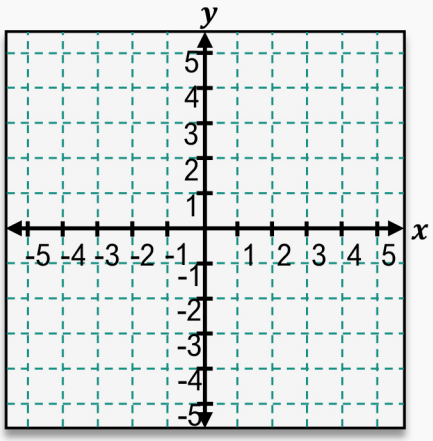
**PRACTICE** Convert the points to polar coordinates

$(\boldsymbol{A})$ $(0,5)$

$(\boldsymbol{B})$ $(-2,2)$

$(\boldsymbol{C})$ $(1,1)$

$(\boldsymbol{D})$ $(-1, -\sqrt{3})$




**HOW TO: Convert Points Rect. to Polar**

**1)** Plot the point $(x, y)$

**2)** Calculate $r = \sqrt{x^2 + y^2}$

**3)** Calculate $\theta$. If point is…

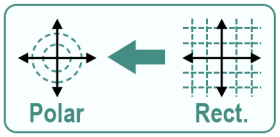
On axis: $\theta = [\,0 \mid \frac{\pi}{2} \mid \pi \mid \frac{3\pi}{2}\,]$

In Q1 or Q4: $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

In Q2 or Q3: $\theta = \tan^{-1}\left(\frac{y}{x}\right) + \pi$

## TOPIC: POLAR COORDINATES

## Convert Equations from Rectangular to Polar Form




- To convert eq'ns from rect. to polar, replace $x$ & $y$ with $r\cos\theta$ & $r\sin\theta$, and solve for ___.
  - Note: You can replace _____ with ___.

**EXAMPLE** Convert each equation to polar form.

**Recall**

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$(\boldsymbol{A})$ $y = 5$

$(\boldsymbol{B})$ $y = x + 1$

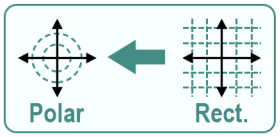
$(\boldsymbol{C})$ $x^2 + y^2 = 25$

**EXAMPLE** Convert each equation to its polar form. In $(\boldsymbol{B})$, solve for $r^2$.

$(\boldsymbol{A})$ $y = x^2$

$(\boldsymbol{B})$ $4xy = 2$

## TOPIC: POLAR COORDINATES




**PRACTICE** Convert each equation to its polar form.

**Recall**

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$(\boldsymbol{A})$

$y - x = 6$

---

$(\boldsymbol{B})$

$3y - 5x = 2$

---

$(\boldsymbol{C})$

$x^2 + y^2 = 2y$

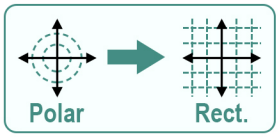
---

$(\boldsymbol{D})$

$x^2 + (y-2)^2 = 4$

## TOPIC: POLAR COORDINATES

## Convert Equations from Polar to Rectangular Form




◆ To convert from polar to rect., manipulate eqn to obtain $r\cos\theta$, $r\sin\theta$, or $r^2$, then replace with $x, y$, or $x^2 + y^2$.

**EXAMPLE** Convert each eqn to rectangular form and identify the shape of its graph.

$(\boldsymbol{A})$ $r = 4$

$(\boldsymbol{B})$ $r = \sec\theta$

$(\boldsymbol{C})$ $r = 6\sin\theta$

**HOW TO: Convert Equations from Polar to Rect.**

**1)** Get $r\cos\theta$, $r\sin\theta$, or $r^2$. Strategies:

- _____ /multiply *both* sides by ___
- Rewrite trig fcns in terms of _____ & _____
- Eliminate fractions by mult. *both* sides by _____

**2)** Replace $r\cos\theta$, $r\sin\theta$, or $r^2$

**3)** Rewrite equation in its "standard form"

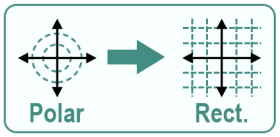
- If $x^2 + y^2 = \# \cdot x$ or $\# \cdot y$, complete the square
- If $\sqrt{\ }$, eliminate by squaring *both* sides

**Recall**

$x = r\cos\theta \qquad y = r\sin\theta$

$x^2 + y^2 = r^2$

## TOPIC: POLAR COORDINATES




**EXAMPLE** Convert the equation to rectangular form and identify the shape of its graph.

$$r = \frac{2}{1 + \cos\theta}$$

**HOW TO: Convert Equations from Polar to Rect.**

**1)** Get $r\cos\theta$, $r\sin\theta$, or $r^2$. Strategies:

- Square/multiply *both* sides by $r$
- Rewrite trig fcns in terms of $\sin$ & $\cos$
- Eliminate fractions by mult. *both* sides by denom.

**2)** Replace $r\cos\theta$, $r\sin\theta$, or $r^2$

**3)** Rewrite equation in its "standard form"

- If $x^2 + y^2 = \# \cdot x$ or $\# \cdot y$, complete the square
- If $\sqrt{\ }$, eliminate by squaring *both* sides

**Recall**

$x = r\cos\theta \qquad y = r\sin\theta$

$x^2 + y^2 = r^2$

**PRACTICE** Convert each equation to its rectangular form.

$(\boldsymbol{A})$

$$r = -4\cos\theta$$

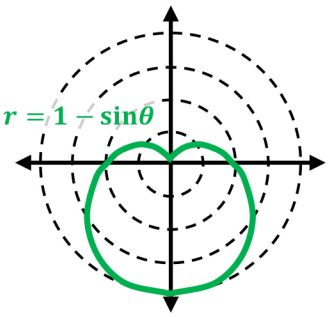
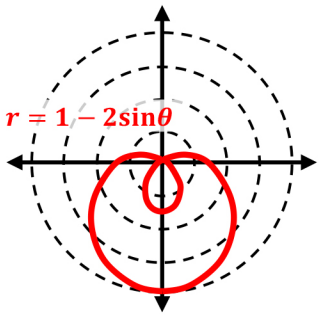
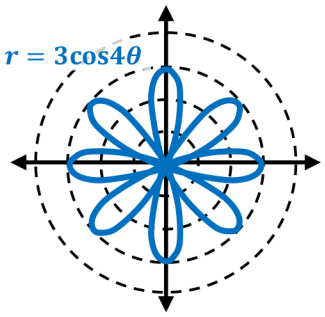
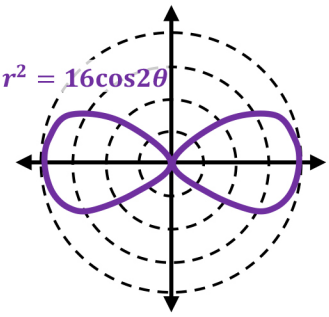
$(\boldsymbol{B})$

$$r = \frac{2}{1 - \sin\theta}$$

## TOPIC: POLAR COORDINATES

### Introduction to Common Polar Equations

- When working with polar equations, you need to *classify* and *graph* these four shapes:

| Cardioids | Limaçons | Roses | Lemniscates |
|---|---|---|---|
|  |  |  |  |
| $r = a \pm b\cos\theta$<br>$r = a \pm b\sin\theta$ | $r = a \pm b\cos\theta$<br>$r = a \pm b\sin\theta$ | $r = a\cos\_\_\theta$<br>$r = a\sin\_\_\theta$ | $r \quad = \pm a^2\cos 2\theta$<br>$r \quad = \pm a^2\sin 2\theta$ |
| $a > 0, b > 0,$<br>$a$ ____ $b$ | $a > 0, b > 0,$<br>$a$ ____ $b$ OR $a$ ____ $b$<br>*(dimple)* *(inner loop)* | $a \neq 0$<br>$n = 2, 3, 4 \ldots$ | $a \neq 0$ |

**EXAMPLE** Classify the given equation.

$(A)$

$$r = 1 + \cos\theta$$

**[ CARDIOID | LIMACON | ROSE | LEMNISCATE ]**

$(B)$

$$r = 4\sin 2\theta$$

**[ CARDIOID | LIMACON | ROSE | LEMNISCATE ]**

**PRACTICE** Identify whether the given equation is that of a cardioid, limaçon, rose, or lemniscate.

$(A)$ $r = 4\sin 2\theta$ **[ CARDIOID | LIMACON | ROSE | LEMNISCATE ]**

$(B)$ $r = 3 + 2\cos\theta$ **[ CARDIOID | LIMACON | ROSE | LEMNISCATE ]**

$(C)$ $r^2 = -25\cos 2\theta$ **[ CARDIOID | LIMACON | ROSE | LEMNISCATE ]**

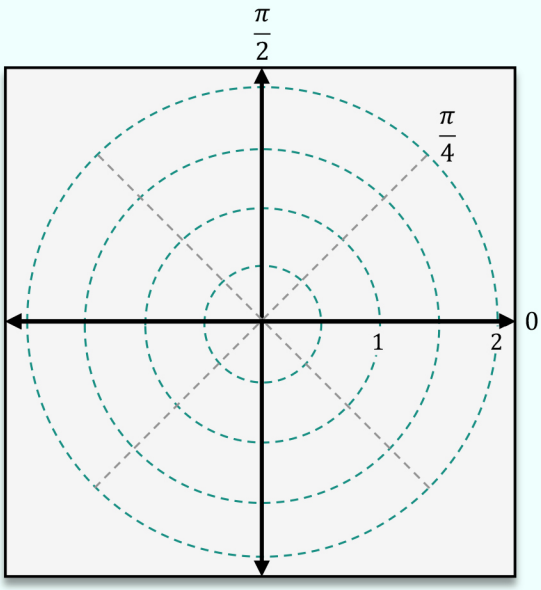
$(D)$ $r = 1 - \sin\theta$ **[ CARDIOID | LIMACON | ROSE | LEMNISCATE ]**

## TOPIC: POLAR COORDINATES

### Cardioids

- Recall: A cardioid is of the form: $r = a \pm b\cos\theta$ OR $r = a \pm b\sin\theta$ where $a = b$.
  - To graph a cardioid, first determine **symmetry**, then plot **points** at quadrantal angles.

**EXAMPLE** Graph $r = 1 + \cos\theta$




**HOW TO: Graph a Cardioid**

**1)** **Symmetry** → If eqn contains:

| cos | sin |
|---|---|
| polar axis | $\theta = \frac{\pi}{2}$ |

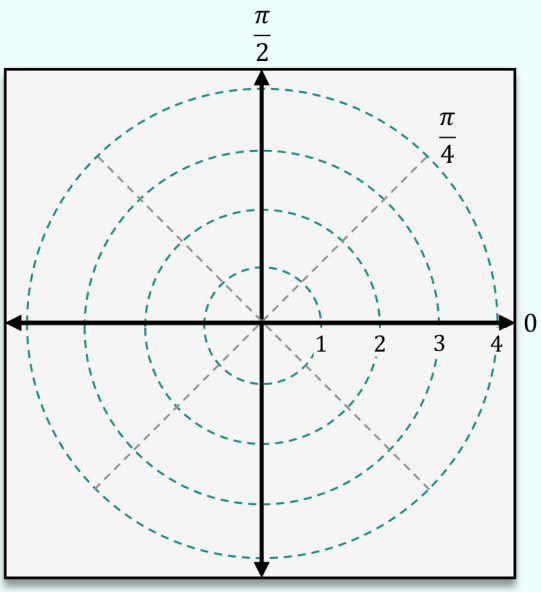
**2)** Find & plot **points** at quadrantal angles

| $\boldsymbol{\theta}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ |
|---|---|---|---|---|
| $\boldsymbol{r}$ | | | | |

**3)** **Connect** with smooth, continuous curve

## TOPIC: POLAR COORDINATES

**EXAMPLE** Graph $r = 2 + 2\sin\theta$




**HOW TO: Graph a Cardioid**

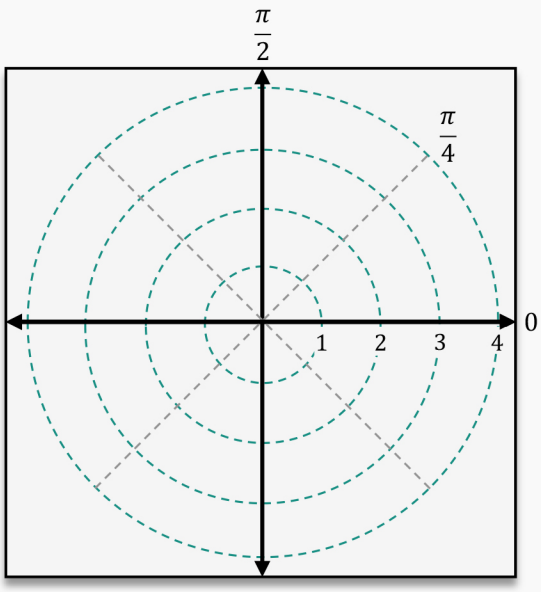
1) **Symmetry** → If eqn contains: $\left[\begin{array}{c|c} \cos & \sin \\ \text{polar axis} & \theta = \frac{\pi}{2} \end{array}\right]$

2) Find & plot **points** at quadrantal angles

| $\boldsymbol{\theta}$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ |
|---|---|---|---|---|
| $\boldsymbol{r}$ | | | | |

3) **Connect** with smooth, continuous curve

**PRACTICE** Graph $r = 2 - 2\cos\theta$




**HOW TO: Graph a Cardioid**

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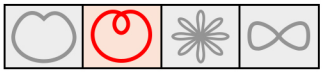
|---|---|---|---|---|
| $\boldsymbol{r}$ | | | | |

3) **Connect** with smooth, continuous curve

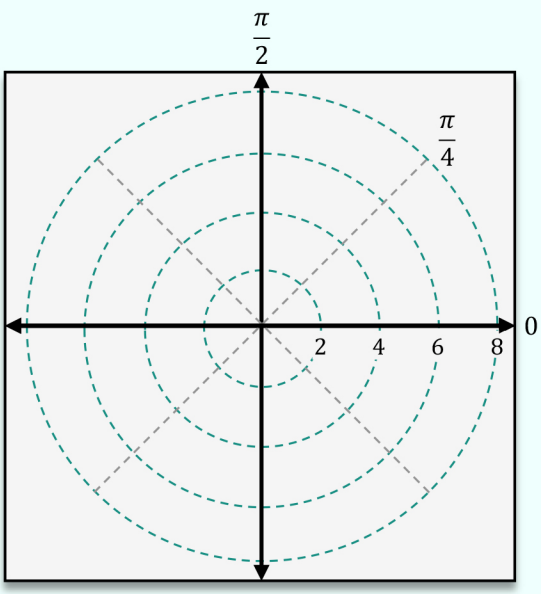
## TOPIC: POLAR COORDINATES

### Limaçons



- Recall: A limaçon is of the form: $r = a \pm b\cos\theta$ OR $r = a \pm b\sin\theta$ where $a > b$ OR $a < b$.
  - Graph limaçons just like cardioids, but first determine if it has a **dimple** or **inner loop**.

**EXAMPLE** Graph $r = 3 - 2\sin\theta$




**HOW TO: Graph a Limaçon**

**1)** If $a$ [ > | < ] $b$, limaçon has [ **dimple** / **no zero** | **inner loop** / **zero** ]

**2)** **Symmetry** → If eqn contains: [ cos / polar axis | sin / $\theta = \frac{\pi}{2}$ ]

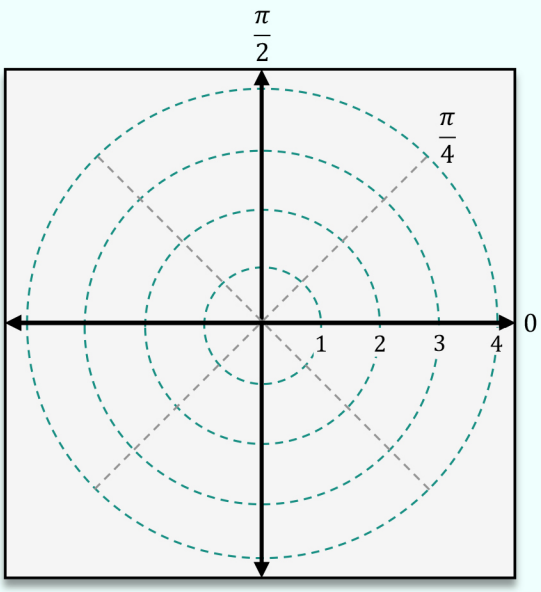
**3)** Find & plot **points** at quadrantal angles

| $\theta$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ |
|---|---|---|---|---|
| $r$ | | | | |

**4)** **Connect** with smooth, continuous curve

## TOPIC: POLAR COORDINATES

**EXAMPLE** Graph $r = 1 + 2\cos\theta$




**HOW TO: Graph a Limaçon**

**1)** If $a$ [ > | < ] $b$, limaçon has [ **dimple** / **no zero** | **inner loop** / **zero** ]

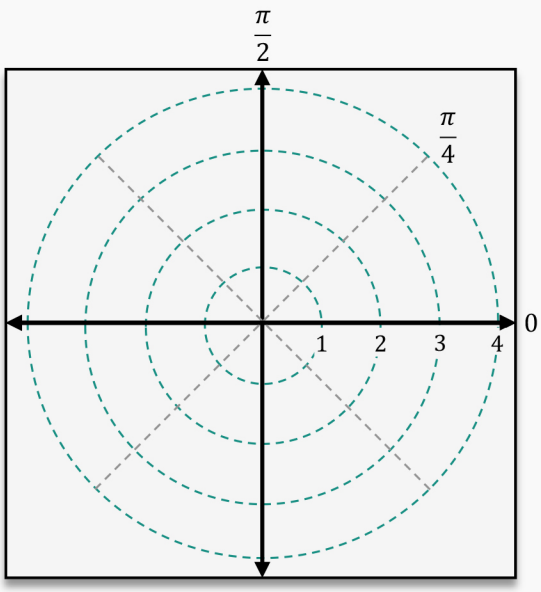
**2)** **Symmetry** → If eqn contains: [ $\cos$ / polar axis | $\sin$ / $\theta = \frac{\pi}{2}$ ]

**3)** Find & plot **points** at quadrantal angles

| $\theta$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ |
|---|---|---|---|---|
| $r$ | | | | |

**4)** **Connect** with smooth, continuous curve

**PRACTICE** Graph $r = 1 + 2\sin\theta$




**HOW TO: Graph a Limaçon**

**1)** If $a$ [ > | < ] $b$, limaçon has [ **dimple** / **no zero** | **inner loop** / **zero** ]

**2)** **Symmetry** → If eqn contains: [ $\cos$ / polar axis | $\sin$ / $\theta = \frac{\pi}{2}$ ]

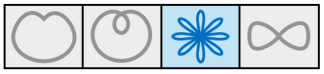
**3)** Find & plot **points** at quadrantal angles

| $\theta$ | $0$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ |
|---|---|---|---|---|
| $r$ | | | | |

**4)** **Connect** with smooth, continuous curve

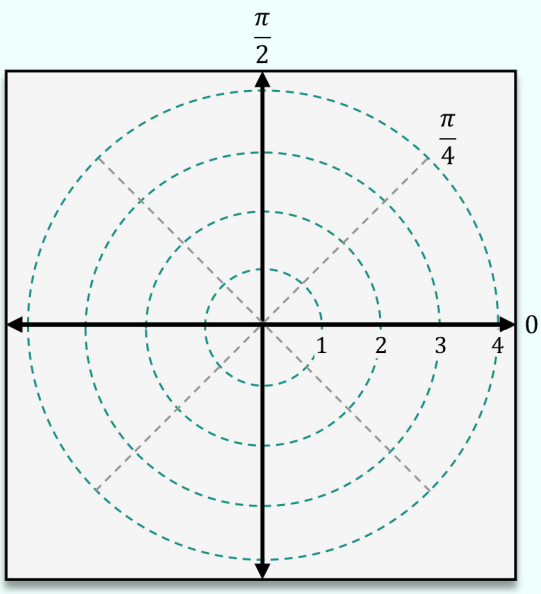
## TOPIC: POLAR COORDINATES

### Roses



- Recall: A rose is of the form: $r = a\cos n\theta$ OR $r = a\sin n\theta$ where $a \neq 0, n \geq 2$.
  - To graph a rose, plot **first petal**, then determine the spacing of other **petals**.

**EXAMPLE** Graph $r = 4\cos 2\theta$




**HOW TO: Graph a Rose**

**1)** For $n$ [**even** | **odd**], [$2n$ | $n$] **petals**: _____

**2)** Plot **first petal**: (_____,_____)
$a$

| cos | sin |
|---|---|
| $\theta = 0$ | $\theta = \frac{\pi}{2n}$ |

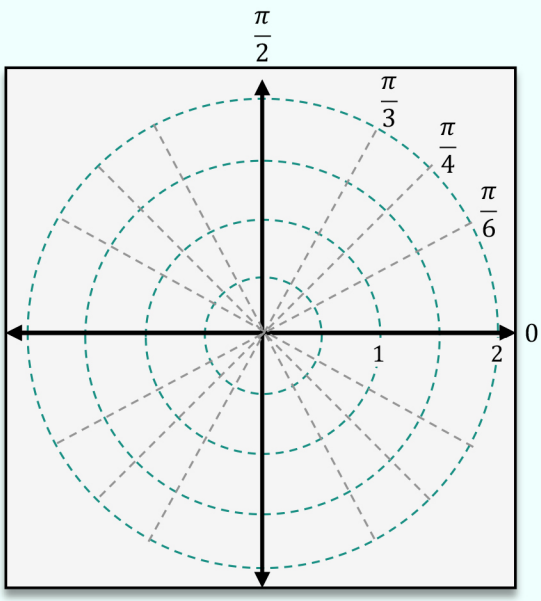
**3)** Plot other **petals** spaced out by $\frac{2\pi}{\#\text{ petals}}$: $\frac{2\pi}{\quad}$

| $\theta$ | | | | |
|---|---|---|---|---|
| $r$ | | | | |

**4)** **Connect** with smooth, continuous curve

## TOPIC: POLAR COORDINATES

**EXAMPLE** Graph $r = 2\sin 3\theta$




**HOW TO: Graph a Rose**

1) For $n$ [**even** | **odd**], [$2n$ | $n$] **petals**: _____

2) Plot **first petal**: (____,____)
$a$

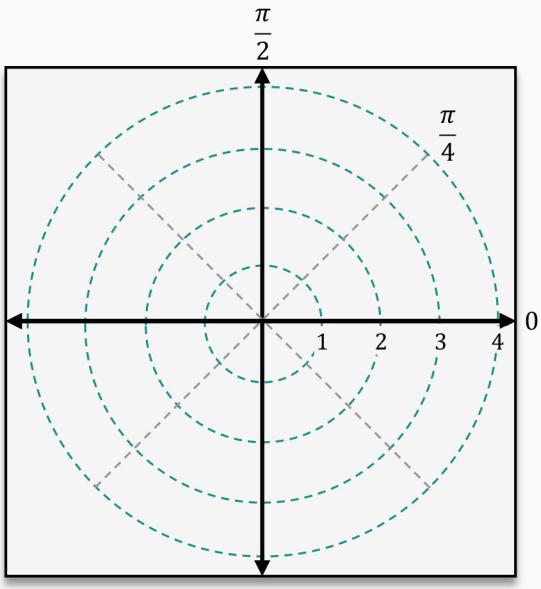
| cos | sin |
|---|---|
| $\theta = 0$ | $\theta = \frac{\pi}{2n}$ |

3) Plot other **petals** spaced out by $\frac{2\pi}{\#\ \text{petals}}$: $\frac{2\pi}{\quad}$

| $\theta$ | | | |
|---|---|---|---|
| $r$ | | | |

4) **Connect** with smooth, continuous curve

**PRACTICE** Graph $r = 3\cos 4\theta$




**HOW TO: Graph a Rose**

1) For $n$ [**even** | **odd**], [$2n$ | $n$] **petals**: _____

2) Plot **first petal**: (____,____)
$a$

| cos | sin |
|---|---|
| $\theta = 0$ | $\theta = \frac{\pi}{2n}$ |

3) Plot other **petals** spaced out by $\frac{2\pi}{\#\ \text{petals}}$: $\frac{2\pi}{\quad}$

| $\theta$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $r$ | | | | | | | | |

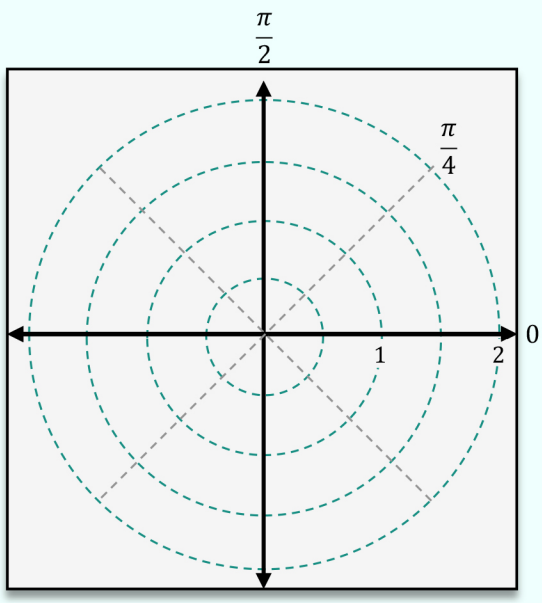
4) **Connect** with smooth, continuous curve

## TOPIC: POLAR COORDINATES

### Lemniscates

- Recall: A lemniscate is of the form: $r^2 = \pm a^2 \cos 2\theta$ OR $r^2 = \pm a^2 \sin 2\theta$ where $a \neq 0$.
  - To graph a lemniscate, plot the **first petal**, then reflect it over the pole.

**EXAMPLE** Graph $r^2 = 4 \sin 2\theta$




**HOW TO: Graph a Lemniscate**

**1)** Plot **first petal**: (____,____)
$\boldsymbol{a}$

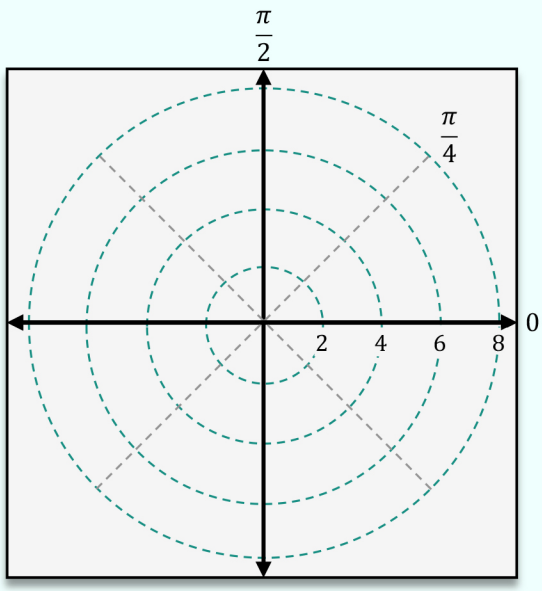
| $+a^2\cos$ | $-a^2\cos$ | $+a^2\sin$ | $-a^2\sin$ |
|---|---|---|---|
| $\theta = 0$ | $\theta = \frac{\pi}{2}$ | $\theta = \frac{\pi}{4}$ | $\theta = \frac{3\pi}{4}$ |

**2)** Reflect **petal** over pole

**3)** **Connect** with smooth, continuous curve

## TOPIC: POLAR COORDINATES

**EXAMPLE** Graph $r^2 = 25\cos 2\theta$




**HOW TO: Graph a Lemniscate**

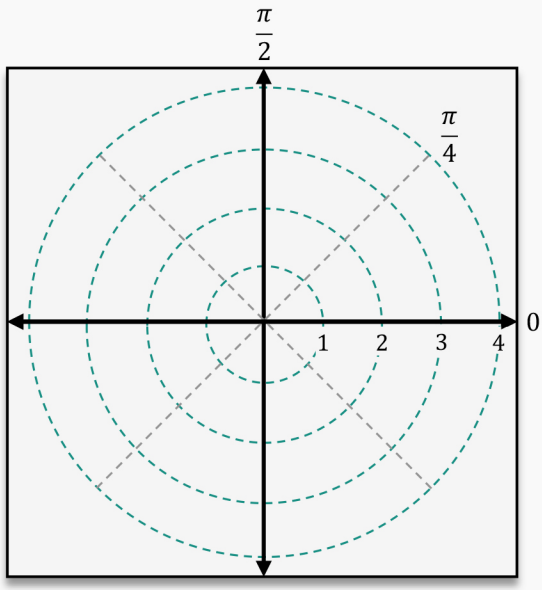
**1)** Plot **first petal**: (____,____)
$a$

| | | | |
|---|---|---|---|
| $+a^2\cos$ | $-a^2\cos$ | $+a^2\sin$ | $-a^2\sin$ |
| $\theta = 0$ | $\theta = \frac{\pi}{2}$ | $\theta = \frac{\pi}{4}$ | $\theta = \frac{3\pi}{4}$ |

**2)** Reflect **petal** over pole

**3)** **Connect** with smooth, continuous curve

**PRACTICE** Graph $r^2 = 9\sin 2\theta$




**HOW TO: Graph a Lemniscate**

**1)** Plot **first petal**: (____,____)
$a$

| | | | |
|---|---|---|---|
| $+a^2\cos$ | $-a^2\cos$ | $+a^2\sin$ | $-a^2\sin$ |
| $\theta = 0$ | $\theta = \frac{\pi}{2}$ | $\theta = \frac{\pi}{4}$ | $\theta = \frac{3\pi}{4}$ |

**2)** Reflect **petal** over pole

**3)** **Connect** with smooth, continuous curve