TOPIC: HYPOTHESIS TESTS FOR PROPORTION

Performing a Hypothesis Test for Population Proportion

- Recall: To run a hypothesis test 1) Write Hypotheses, 2) Calc. Test Statistic, 3) Find P-Value, & 4) State Conclusion.

EXAMPLE

A tech company says 90% of its devices pass inspection. A quality inspector thinks it's less, so they test 200 devices, 172 of which passed. At the 0.01 significance level, is there evidence the pass rate is below 90%?

New — **Hypothesis Tests for Proportion**

1) Hyp

$H_0: p =$ ______ $\qquad H_a: p$ **[< | > | ≠]**

2) Test Stat

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$

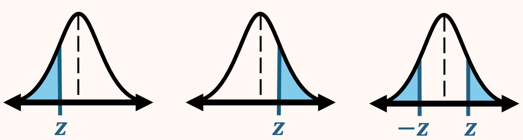
$n =$ _____ $\quad x =$ _____ $\quad \hat{p} = \frac{x}{n} =$ _____

$z =$

3) P-Value

Area "beyond" z

If $H_a: p <$ $\qquad$ If $H_a: p >$ $\qquad$ If $H_a: p \neq$



P-Value = ______

4) Conclusion

Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0.

There is **[ENOUGH | NOT ENOUGH]** evidence to suggest…

Criteria

Random Samples? ☐

$np \geq 5$ ☐

$nq \geq 5$ ☐

Recall: $q = 1 - p$

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EXAMPLE Perform each hypothesis test using $n = 20$, $x = 14$, a claim of $p = 0.5$ & $\alpha = 0.05$.

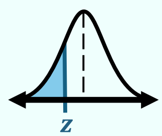
($\boldsymbol{A}$) One tail - Left

Recall

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$

$$\hat{p} = \frac{x}{n}, \qquad q = 1 - p$$

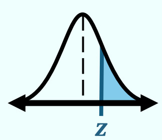
Random Samples?	☐	H_0:	H_a:	
$np \geq 5$	☐	$n =$ ___	$x =$ ___	$\hat{p} =$ ___
$nq \geq 5$	☐	$z =$		



Because P-value = ___ **[< | >]** α = ____ , we **[REJECT | FAIL TO REJECT]** H_0.
There is **[ENOUGH | NOT ENOUGH]** evidence to suggest $H_a: p$ **[< | > | ≠]** 0.5

($\boldsymbol{B}$) One tail - Right

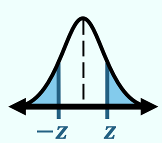
Random Samples?	☐	H_0:	H_a:	
$np \geq 5$	☐	$n =$ ___	$x =$ ___	$\hat{p} =$ ___
$nq \geq 5$	☐	$z =$		



Because P-value = ___ **[< | >]** α = ____ , we **[REJECT | FAIL TO REJECT]** H_0.
There is **[ENOUGH | NOT ENOUGH]** evidence to suggest $H_a: p$ **[< | > | ≠]** 0.5

($\boldsymbol{C}$) Two tail

Random Samples?	☐	H_0:	H_a:	
$np \geq 5$	☐	$n =$ ___	$x =$ ___	$\hat{p} =$ ___
$nq \geq 5$	☐	$z =$		



Because P-value = ___ **[< | >]** α = ____ , we **[REJECT | FAIL TO REJECT]** H_0.
There is **[ENOUGH | NOT ENOUGH]** evidence to suggest $H_a: p$ **[< | > | ≠]** 0.5

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PRACTICE Perform a 2-tailed hypothesis test for the true proportion of successes using the given values:

Recall

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$

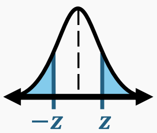
$$\hat{p} = \frac{x}{n}, \qquad q = 1 - p$$

$(\boldsymbol{A})$ $\alpha = 0.01$, $n = 40$, $x = 28$, & claim is $p = 0.75$

Random Samples? ☐ H_0: H_a:

$np \geq 5$ ☐ $\hat{p} =$ ___ $z =$

$nq \geq 5$ ☐



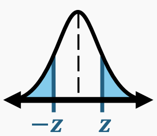
Because P-value = ___ **[< | >]** α = ____ , we **[REJECT | FAIL TO REJECT]** H_0.
There is **[ENOUGH | NOT ENOUGH]** evidence to suggest $H_a: p$ **[< | > | ≠]** 0.75

$(\boldsymbol{B})$ $\alpha = 0.10$, $n = 100$, $x = 42$, & claim is $p = 0.25$

Random Samples? ☐ H_0: H_a:

$np \geq 5$ ☐ $\hat{p} =$ ___ $z =$

$nq \geq 5$ ☐



Because P-value = ___ **[< | >]** α = ____ , we **[REJECT | FAIL TO REJECT]** H_0.
There is **[ENOUGH | NOT ENOUGH]** evidence to suggest $H_a: p$ **[< | > | ≠]** 0.25

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EXAMPLE

A company claims that 20% of its products are defective. A random sample of 150 products is tested, & 35 are found defective. At $\alpha = 0.05$, test whether the defect rate is different from 20%. Based on your results, should the company investigate its production process?

Random Samples? ☐ H_0: H_a:

$np \geq 5$ ☐ $n =$ ___ $x =$ ___ $\hat{p} =$ ___

$nq \geq 5$ ☐ $z =$

Recall

$$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$$

$$\hat{p} = \frac{x}{n}, \qquad q = 1 - p$$

Because P-value = ___ **[< | >]** α = ____ , we **[REJECT | FAIL TO REJECT]** H_0.
There is **[ENOUGH | NOT ENOUGH]** evidence to suggest…

PRACTICE

A snack company claims that at least 70% of people prefer its new low-sugar granola bar over the original version. To test this claim, a grocery chain surveys a random sample of 80 customers, & 50 say they prefer the new version. Use $\alpha = 0.10$ to test whether more than 70% of customers prefer the new granola bar. Should the grocery chain stock more of the new product & reduce shelf space for the original version?

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<table>
<tr><th></th><th colspan="2">Mean</th><th>Proportion</th><th>Variance</th></tr>
<tr><td>Step 1) Hypotheses</td><td colspan="2">$H_0: \mu =$ ___
$H_a: \mu$ $\mathbf{[\, \lt \mid \gt \mid \neq \,]}$ ___</td><td>$H_0: p =$ ___
$H_a: p$ $\mathbf{[\, \lt \mid \gt \mid \neq \,]}$ ___</td><td>$H_0: \sigma^2 =$ ___
$H_a: \sigma^2$ $\mathbf{[\, \lt \mid \gt \mid \neq \,]}$ ___</td></tr>
<tr><td>Step 2) Calc. Test Statistic</td><td>(σ Known):
$z = \frac{x - \mu}{\sigma/\sqrt{n}}$</td><td>(σ Unknown):
$t = \frac{x - \mu}{s/\sqrt{n}}$</td><td>$z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}$</td><td>$\mathcal{X}^2 = \frac{(n-1)s^2}{\sigma^2}$</td></tr>
<tr><td>Step 3) P-Value</td><td>$P(Z\ \mathbf{[\, \lt \mid \gt \,]}\ z)$ OR
$2 \cdot P(Z\ \mathbf{[\, \lt \mid \gt \,]}\ z)$</td><td>$P(T_{n-1}\ \mathbf{[\, \lt \mid \gt \,]}\ t)$ OR
$2 \cdot P(T_{n-1}\ \mathbf{[\, \lt \mid \gt \,]}\ t)$</td><td>$P(Z\ \mathbf{[\, \lt \mid \gt \,]}\ z)$ OR
$2 \cdot P(Z\ \mathbf{[\, \lt \mid \gt \,]}\ z)$</td><td>$P(\mathcal{X}^2_{n-1}\ \mathbf{[\, \lt \mid \gt \,]}\ \mathcal{X}^2)$ OR
$2 \cdot P(\mathcal{X}^2_{n-1}\ \mathbf{[\, \lt \mid \gt \,]}\ \mathcal{X}^2)$</td></tr>
<tr><td>Step 4) State Conclusion</td><td colspan="4">Because P-value $\mathbf{[\, \lt \mid \gt \,]}$ α, we [REJECT | FAIL TO REJECT] H_0.
There is [ENOUGH | NOT ENOUGH] evidence to { restate H_a }</td></tr>
</table>