TOPIC: TWO PROPORTIONS

Difference in Proportions: Hypothesis Tests

◆ In hypothesis tests with ***TWO*** samples, we test claims about the ____________ between the proportions.

- ► Write H_0 as $p_1 =$ _______, i.e. $p_1 - p_2 =$ ____
- ► Find the z-score using a *pooled* sample proportion ($\bar{p}$) which is $\frac{total\ \#}{total\ \#}$ ____________ of both groups.

EXAMPLE

The table summarizes a study on the success rate of a nicotine patch in helping people quit smoking. Perform a hypothesis test using $\boldsymbol{\alpha = 0.05}$ to determine if the proportion of subjects who quit smoking is different in the two groups.

Effectiveness of Nicotine Patch	
Placebo	**Nicotine Patch**
Subjects: 20	Subjects: 23
# Successfully Quit: 11	# Successfully Quit: 17

Samples Random and Independent? ☐

≥ 5 success in each sample? ☐

≥ 5 failures in each sample? ☐

1) H_0:

H_a:

Placebo **Patch**

2) $n_1 =$ ___ $n_2 =$ ___

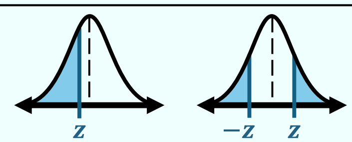
$x_1 =$ ___ $x_2 =$ ___

$\hat{p}_1 =$ ___ $\hat{p}_2 =$ ___

$\bar{p} =$

$\bar{q} =$

$z =$

<table>
<tr><th></th><th>1 Prop.</th><th>2 Proportions</th></tr>
<tr><td>Hyp.</td><td>$H_0: p = \#$
$H_a: p$ </>/≠ #</td><td>$H_0: p_1 = p_2$
$H_a: p_1$ [< | > | ≠] p_2</td></tr>
<tr><td>Test Stat.</td><td>$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$</td><td>$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$
$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$
$\bar{q} = 1 - \bar{p}$</td></tr>
<tr><td>P-Val.</td><td colspan="2">Area "beyond" z 
</td></tr>
<tr><td>Conclude</td><td colspan="2">Because P-value [< | >] α, we
[REJECT | FAIL TO REJECT] H_0. There is
[ENOUGH | NOT ENOUGH] evidence to { restate H_a }</td></tr>
</table>

3) P-value:

4) Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0.
There is **[ENOUGH | NOT ENOUGH]** evidence that there is a difference in proportion of people quitting smoking using the nicotine patch versus a placebo.

TOPIC: TWO PROPORTIONS

PRACTICE

A human resources department is comparing two employee training programs to see if they lead to different pass rates on a required certification program. They randomly select two groups of employees. In Program A, 16 out of 20 employees passed the exam. In Program B, 30 out of 40 employees passed. Are the basic conditions met to conduct a 2-proportion hypothesis test?

(A) Yes, the basic conditions are met

(B) No, the basic conditions are not met

(C) There is not enough information to answer the question

PRACTICE

A school administrator wants to compare the proportion of students who passed a standardized math exam in two different schools by taking samples from 2 classes. Assume the samples are random and independent. Calculate the z-score for testing whether there is a significant difference in the population proportions of student passing rates, but do not find a P-value or draw a conclusion for the hypothesis test.

Class A: 72 out of 120 students passed

Class B: 65 out of 100 students passed

TOPIC: TWO PROPORTIONS

EXAMPLE

A study on the effectiveness of seatbelts in reducing injuries is done using two random samples of drivers. Among the group who **were not** wearing their seatbelt, 50 drivers were injured and 2350 were not. Among the group who **were** wearing a seatbelt, 15 were injured and 1585 were not. Use a 0.01 significance level to test the claim that not wearing a seatbelt results in a greater proportion of injuries.

TOPIC: TWO PROPORTIONS

Difference in Proportions: Confidence Intervals

◆ To make a Conf. Int. for $p_1 - p_2$, use point estimator $\hat{p}_1 - \hat{p}_2$ and margin of error:

- ► Unlike Hyp. Tests, use *individual* sample proportions instead of *pooled*.

New

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\quad}{n_1} + \frac{\quad}{n_2}}$$

EXAMPLE

The table summarizes a study on the success rate of nicotine patch in helping people quit smoking. Make a 90% confidence interval for the difference in success rates for the two groups. What does the result suggest about the claim that there is no difference in proportion between the two groups?

Effectiveness of Nicotine Patch	
Placebo	**Nicotine Patch**
Subjects: 20	Subjects: 23
# Who Quit: 11	# Who Quit: 17
$\hat{p}_1 = 0.55$	$\hat{p}_2 = 0.74$

We are ______ % confident that the true difference in proportions of people who quit smoking with a placebo vs. a nicotine patch is between ________ & ________. Because this **[DOES | DOES NOT]** include 0, we **[REJECT | FAIL TO REJECT]** $H_0: p_1 = p_2$. There is **[ENOUGH | NOT ENOUGH]** evidence that there is a difference...

HOW TO: Make a Confidence Interval for $p_1 - p_2$

1) Verify for EACH sample:

# of successes ≥ 5 ☐

AND # of failures ≥ 5 ☐

2) Find critical value: $z_{\alpha/2}$

3) Point estimate: $\hat{p}_1 - \hat{p}_2$

4) Margin of Error: E

5) Find upper & lower bounds

$\big((\hat{p}_1 - \hat{p}_2) - E, (\hat{p}_1 - \hat{p}_2) + E\big)$

Recall

$\hat{p}_1 = \frac{x_1}{n_1}, \quad \hat{p}_2 = \frac{x_2}{n_2}$

◆ If Conf. Int. *DOESN'T* include 0, we're confident of a *DIFFERENCE* between p_1 & p_2, so we ____________ H_0.

◆ If Conf. Int. *DOES* include 0, it's possible there is *NO DIFFERENCE* between p_1 & p_2, so we ____________ H_0.

TOPIC: TWO PROPORTIONS

PRACTICE

A researcher using a survey constructs a 90% confidence interval for a difference in two proportions. According to the data, they calculate $\hat{p}_1 - \hat{p}_2 = 0.09$ with a margin of error of 0.07. Should they reject or fail to reject the claim that there is no difference in these two proportions?

(A) Reject

(B) Fail to reject

(C) There is not enough information to answer the question

PRACTICE

The data below is taken from two random, independent samples. Calculate the margin of error for a 99% confidence interval for the difference in population proportions.

$x_1 = 87, \quad x_2 = 68$

$n_1 = 120, \quad n_2 = 115$

TOPIC: TWO PROPORTIONS

EXAMPLE

A university wants to know if students who live on campus are more likely to attend campus events than students who live off campus. In a sample of 150 **on-campus** students, 102 attended at least one campus event in the past month. In a sample of 130 **off-campus** students, 74 attended at least one campus event in the past month. Construct a 95% confidence interval for the difference in proportion, and use it to test the claim that on-campus students are more likely to attend events than off-campus students.

HOW TO: Make a Confidence Interval for $p_1 - p_2$

1) Verify for EACH sample:

# of successes ≥ 5 ☐

AND # of failures ≥ 5 ☐

2) Find critical value: $z_{\alpha/2}$

3) Point estimate: $\hat{p}_1 - \hat{p}_2$

4) Margin of Error: E

5) Find upper & lower bounds

$\big((\hat{p}_1 - \hat{p}_2) - E, (\hat{p}_1 - \hat{p}_2) + E\big)$

TOPIC: TWO PROPORTIONS

Two Proportion Inferences Using a Calculator

◆ To perform a Hypothesis Test for 2 population proportions using a calculator, use the **`2-PropZTest`** function.

EXAMPLE

For $x_1 = 186$, $n_1 = 200$, $x_2 = 211$, $n_2 = 250$ & $\alpha = 0.05$, test the claim that $p_1 = p_2$ using a Hyp. Test.

H_0:__________

H_a:__________

P-value: __________

Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0, there is **[ENOUGH | NOT ENOUGH]** evidence to suggest…

HOW TO: Hyp. Test for 2 Props (TI-84)

1) STAT > **TESTS**

`6:2-PropZTest`

2) **`x1:`**

`n1:`

`x2:`

`n2:`

`p1: ≠p2 <p2 >p2`

`Calculate Draw`

◆ To make a C.I. for 2 population proportions using a calculator, use the **`2-PropZInt`** function.

EXAMPLE

For $x_1 = 186$, $n_1 = 200$, $x_2 = 211$, $n_2 = 250$ & $\alpha = 0.05$, test the claim that $p_1 = p_2$ using a C.I.

C-Level: _______

Confidence Interval: (________ , ________)

We are ____% sure the diff. btw. the 2 pop. proportions is between ___ & ___. The int. **[DOES | DOESN'T]** include ___, so there's **[ENOUGH | NOT ENOUGH]** evidence to suggest…

HOW TO: Hyp. Test for 2 Props (TI-84)

1) STAT > **TESTS**

`B:2-PropZInt`

2) **`x1:`**

`n1:`

`x2:`

`n2:`

`C-Level:`

`Calculate`

TOPIC: TWO PROPORTIONS

PRACTICE For $x_1 = 34$, $n_1 = 50$, $x_2 = 52$, & $n_2 = 75$, test the claim that $p_1 < p_2$ for $\alpha = 0.01$ using…

$(\boldsymbol{A})$ A Hypothesis Test.

$(\boldsymbol{B})$ Confidence Interval.

HOW TO: Hyp. Test for 2 Props (TI-84)

1) STAT > **TESTS**

   6:2-PropZTest

2) x1:
   n1:
   x2:
   n2:
   p1: ≠p2 **<p2** >p2
   Calculate Draw

HOW TO: Conf. Int. for 2 Props (TI-84)

1) STAT > **TESTS**

   B:2-PropZInt

2) x1:
   n1:
   x2:
   n2:
   C-Level:
   Calculate

TOPIC: TWO PROPORTIONS

EXAMPLE

A bike retailer collects data on the proportion of students who commute by bicycle at two universities: University A and University B. Out of 120 students surveyed at University A, 36 commute by bicycle. Out of 150 students surveyed at University B, 51 commute by bicycle.

(*A*) At the 0.05 significance level, test the claim that the proportion of students who commute by bicycle is the same at both universities.

(*B*) Construct a 95% confidence interval for the difference in proportions.

(*C*) The retailer will advertise at both universities if the proportion of bike commuters is the same at each, otherwise, they will devote their advertising budget to the university with the highest proportion of riders. How will they devote their advertising budget?

HOW TO: Hyp. Test for 2 Props (TI-84)

1) STAT > **TESTS**

`6:2-PropZTest`

2) `x1:`
`n1:`
`x2:`
`n2:`
`p1: ≠p2 <p2 >p2`

`Calculate Draw`

HOW TO: Conf. Int. for 2 Props (TI-84)

1) STAT > **TESTS**

`B:2-PropZInt`

2) `x1:`
`n1:`
`x2:`
`n2:`
`C-Level:`

`Calculate`

TOPIC: TWO PROPORTIONS

EXAMPLE

A company is looking to outsource communications to a call center and is trying to decide between two options. They look at some statistics and find that 476 of 500 customers are satisfied for Center A and 623 out of 700 are satisfied for Center B. They are interested in seeing if there is enough evidence to suggest that the proportion of satisfied customers is higher for Center A. For $\alpha = 0.1$, test this with…

$(\boldsymbol{A})$ A Hypothesis Test.

$(\boldsymbol{B})$ A Confidence Interval.

$(\boldsymbol{C})$ Do they have enough evidence to suggest Center A is a better option?

HOW TO: Hyp. Test for 2 Props (TI-84)

1) STAT > **TESTS**

6:2-PropZTest

2) x1:
n1:
x2:
n2:
p1: ≠p2 <p2 **>p2**

Calculate Draw

HOW TO: Conf. Int. for 2 Props (TI-84)

1) STAT > **TESTS**

B:2-PropZInt

2) x1:
n1:
x2:
n2:
C-Level:

Calculate