

**TOPIC: ESTIMATING AREA WITH FINITE SUMS**

**Estimating the Area Under a Curve Using Left Endpoints**

◆ To estimate the area under a curve, break it into many ( $n$ ) \_\_\_\_\_ and add all of their areas.

▶ The height of each rectangle is the *function value* at the left endpoint and the width is  $\Delta x = \frac{b-a}{n}$ .

**EXAMPLE**

Approximate the area under the curve (region  $R$ ) using (A) 2 rectangles and (B) 4 rectangles.

New
Estimating Area Under A Curve Using Left Endpoints

$n = 2$

$f(x) = 4 - x^2$

$a = 0$ ,  $b = 2$ ,  $\Delta x = 1$

$A \approx$

$\approx$

$\approx \cdot + \cdot$

$\approx$

$\approx$

$n = 4$

$f(x) = 4 - x^2$

$a = 0$ ,  $b = 2$ ,  $\Delta x = 0.5$

$A \approx$

$\approx$

$\approx ( + + + )$

$\approx$

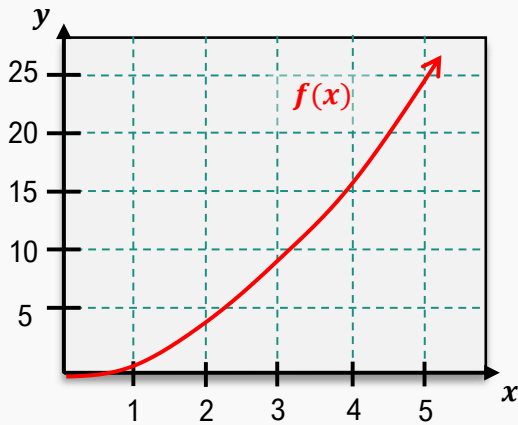
$\approx$

◆ The more rectangles (*subintervals*) we break our region into, the more \_\_\_\_\_ our estimate gets.

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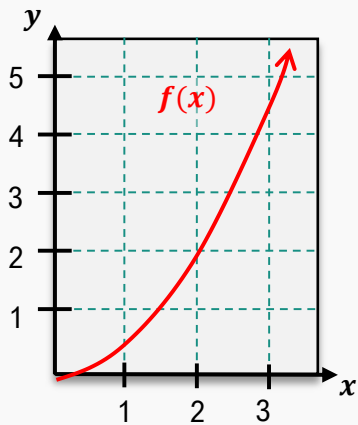
### PRACTICE

Use five rectangles to estimate the area under the curve of  $f(x) = x^2$  from  $x = 0$  to  $x = 5$  using left endpoints.



### PRACTICE

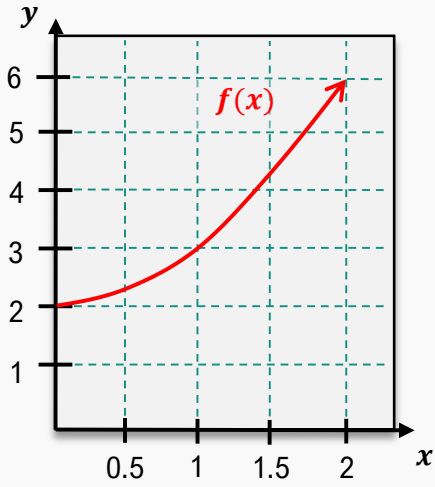
Use two rectangles to estimate the area under the curve of  $f(x) = \frac{1}{2}x^2$  from  $x = 0$  to  $x = 3$  using left endpoints.



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**PRACTICE**

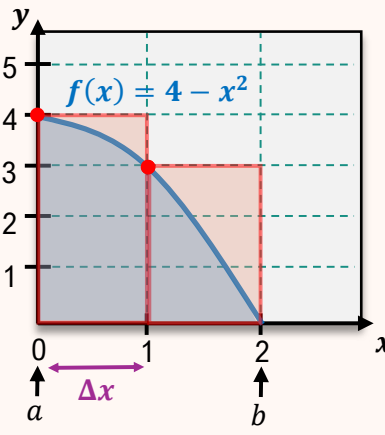
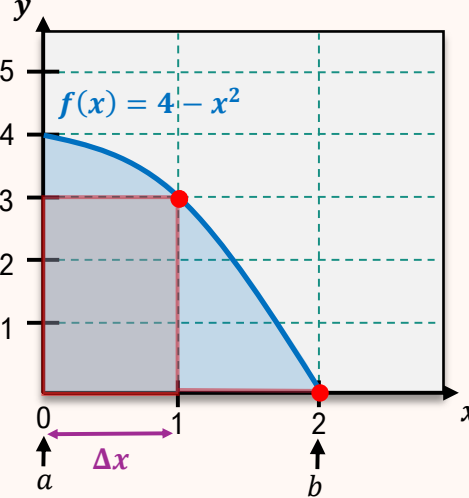
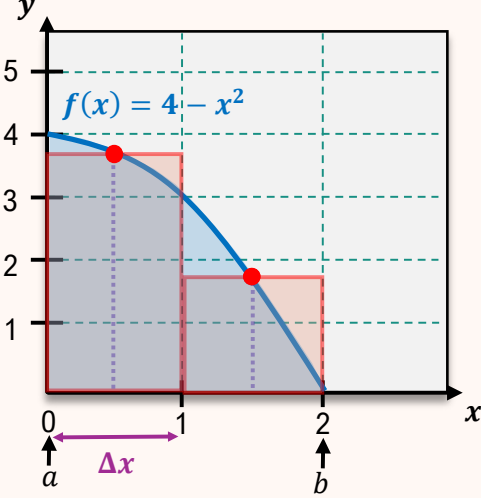
Use four rectangles to estimate the area under the curve of  $f(x) = x^2 + 2$  from  $x = 0$  to  $x = 2$  using left endpoints.



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**Estimating the Area Under a Curve Using Right Endpoints & Midpoints**

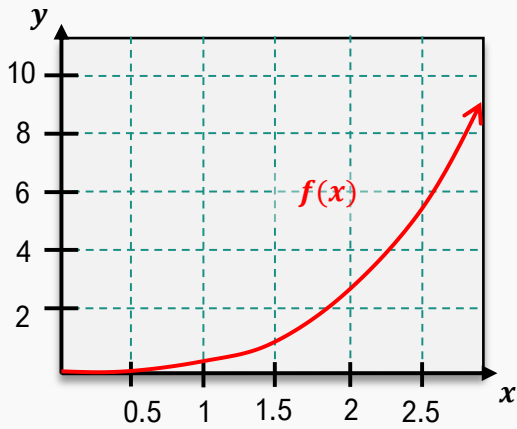
- ◆ Recall: We can estimate the area under a curve by breaking it into rectangles (subintervals) and using left endpts.
  - ▶ We can also use the fcn value at the \_\_\_\_\_ endpoint or \_\_\_\_\_ as the height of each rectangle.

Recall <b>Left Endpt Approx.</b>	New <b>Right Endpt Approx.</b>	New <b>Midpoint Approx.</b>
 <p style="text-align: center;"><math>f(x) = 4 - x^2</math></p> <p style="text-align: center;"><math>\Delta x = \frac{b - a}{n}</math></p> <p><math>A \approx A_1 + A_2</math></p> <p><math>\approx w_1 \cdot h_1 + w_2 \cdot h_2</math></p> <p><math>\approx \Delta x \cdot f(0) + \Delta x \cdot f(1)</math></p>	 <p style="text-align: center;"><math>f(x) = 4 - x^2</math></p> <p><math>A \approx A_1 + A_2</math></p> <p><math>\approx w_1 \cdot h_1 + w_2 \cdot h_2</math></p> <p><math>\approx \Delta x \cdot \quad + \Delta x \cdot \quad</math></p>	 <p style="text-align: center;"><math>f(x) = 4 - x^2</math></p> <p><math>A \approx A_1 + A_2</math></p> <p><math>\approx w_1 \cdot h_1 + w_2 \cdot h_2</math></p> <p><math>\approx \Delta x \cdot \quad + \Delta x \cdot \quad</math></p>

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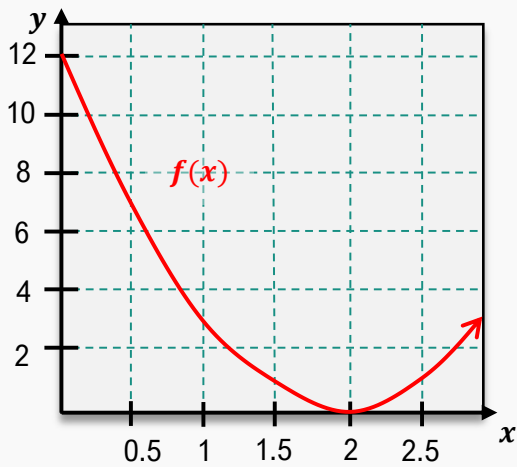
### PRACTICE

Use three rectangles to estimate the area under the curve of  $f(x) = \frac{1}{3}x^3$  from  $x = 0$  to  $x = 3$  using the right endpoints.



### PRACTICE

Use three rectangles to approximate the area under the curve of  $f(x) = 3(x - 2)^2$  from  $x = 0$  to  $x = 3$  using the midpoint rule.



## **TOPIC: ESTIMATING AREA WITH FINITE SUMS**

### **EXAMPLE**

Approximate the area under the curve of  $f(x) = x^3$  on the interval  $[0, 3]$  using 3 rectangles for all the following methods:

- a. Left Endpoints
- b. Right Endpoints
- c. Midpoint rule