## TOPIC: MULTIPLYING AND DIVIDING COMPLEX NUMBERS

### Multiplying Complex Numbers

- Complex numbers are multiplied just like algebraic expressions! We A) ______________ or B) _________
  - Multiplying will *ALWAYS* produce an $\boldsymbol{i}^2$ term that will get simplified.

EXAMPLE: Find the product. Write answers in standard form.

$(\boldsymbol{A})$ $3i(7-2i)$

$(\boldsymbol{B})$ $(-6+2i)(3+4i)$

| MULTIPLYING COMPLEX NUMBERS |
|---|
| **1)** Distribute or FOIL |
| **2)** Apply $\boldsymbol{i}^2=-1$ |
| **3)** Combine Like Terms |

**PRACTICE** Perform the indicated operation. Express your answer in standard form.

$(3+8i)^2$

**PRACTICE** Find the product. Express your answer in standard form.

$2i(9-4i)(6+5i)$

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**PRACTICE** Multiply the following and simplify.

$(\boldsymbol{A})$ $(5 - i)(12)$

$(\boldsymbol{B})$ $(13i)(17i)$

$(\boldsymbol{C})$ $(7 + 3i)(7 - 3i)$

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### Complex Conjugates

◆ Reverse the ________ of *only* the **imaginary** part of a complex number to get the **conjugate:** $a + b\boldsymbol{i} \Leftrightarrow$

EXAMPLE: Find the conjugate of each complex number.

**(A)** $1 + 2i$

**(B)** $1 - 2i$

**(C)** $-1 + 2i$

◆ Multiplying **complex conjugates** (by FOIL) ***ALWAYS*** results in a _________ number

EXAMPLE: Find the product.

$$(2 + 3i)(2 - 3i)$$

$(a + bi)(a - bi) =$ ____________

**PRACTICE** Find the product of the given complex number and its conjugate.

**(A)** $4 - 5i$

**(B)** $-7 - i$

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### Dividing Complex Numbers

$$\frac{c}{a + bi}$$

- Dividing by a complex number results in a fraction with ***i*** in the bottom: this is **BAD**
  - Denominators should ***ALWAYS*** be real! To do this, multiply by its ____________

EXAMPLE: Find the quotient. Write answer in standard form.

$$\frac{3}{1 + 2i}$$

**DIVIDING COMPLEX NUMBERS**

**1)** Multiply top AND bottom by complex conj. of bottom & simplify

**2)** Expand fraction into real & imaginary parts

**3)** Simplify fractions into lowest terms

**PRACTICE** Find the quotient. Express your answer in standard form.

$(\boldsymbol{A})$ $$\frac{6 + i}{4 - 2i}$$

$(\boldsymbol{B})$ $$\frac{-5 + 3i}{-7 - 4i}$$

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**Powers of $\boldsymbol{i}$**

◆ Recall: $\boldsymbol{i} = \sqrt{-\mathbf{1}}$. Many problems will have $\boldsymbol{i}$ raised to the $2^{nd}$, $3^{rd}$, or even much higher powers!

► All properties of exponents can be applied to powers of $\boldsymbol{i}$

| **POWERS OF $\boldsymbol{i}$** | |
|---|---|
| $i^1 =$ ⟶ ____ | $i^5 =$ ______ = ______ = ____ |
| $i^2 =$ ⟶ _____ = ____ | $i^6 =$ ______ = ______ = ____ |
| $i^3 =$ _____ = _____ = ____ | $i^7 =$ ⟶ ____ |
| $i^4 =$ _____ = _____ = ____ | $i^8 =$ ⟶ ____ |

► ***Any*** power of $\boldsymbol{i}$ can ***ALWAYS*** be simplified to ___, ___, ___ or ___

**PRACTICE** Evaluate the following powers of $i$.

$(\boldsymbol{A})$ $(3i)^4$

$(\boldsymbol{B})$ $(4i)^{-3}$

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### How to Evaluate Higher Powers of $i$

◆ We can express powers of $i$ in terms of ______.

EXAMPLE: Simplify the power of $i$.

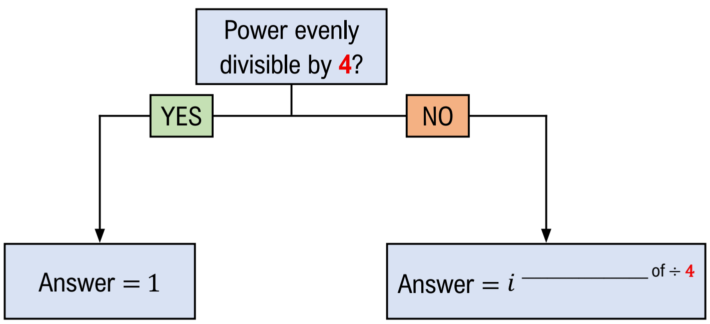
**(A)**

$$i^{20} = i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4$$
$$= 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$$
$$=$$

**(B)**

$$i^{22} = \underbrace{i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4}_{} \cdot i^2$$
$$=$$
$$=$$

◆ To evaluate $i$ raised to a *very high* power, here's a shortcut:




| Remainders of $i$ |
|---|
| $i^1 = i$ |
| $i^2 = -1$ |
| $i^3 = -i$ |

EXAMPLE: Simplify the power of $i$.

**(A)** $i^{100}$

**(B)** $i^{22}$

**(C)** $i^{67}$

**PRACTICE** Simpllify the power of $i$.

**(A)** $i^{1003}$

**(B)** $i^{85}$

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**EXAMPLE** Perform the indicated operation and simplify where possible.

$(\boldsymbol{A})$

$$i^{12} + i^{15}$$

$(\boldsymbol{B})$

$$i^{8} \times i^{5}$$

$(\boldsymbol{C})$

$$\frac{i^{10} + i^{15}}{i^{9} - i^{12}}$$